Determination of NC/CC Ratio and CC shape During the Pure D_2O Stage
(Part II)

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Abstract

A simple model was developed to explain theoretically the observed distribution throughout the vessel of NC events during the pure D_2O stage discussed in our previous report (SNO-STR-98-001). This work also briefly describes the determination of NC/CC ratio and CC shape for various MSW scenarios during this stage. The result suggests that SNO might not miss the advantages of using salt or neutron counters when starting with D_2O runs.

Introduction:

Question: Can we determine the NC/CC ratio and the CC shape during the pure D_2O running stage? Earle and Wong [1] first pointed out, W. Frati and E. Beier also noticed [2] the marked radial distribution of neutron captures in a SNO-vessel containing highly enriched D_2O. In our previous report [3], we briefly described a method of determining the NC/CC ratio using the fact that the observed neutral current events have a non-uniform spatial distribution throughout the acrylic vessel. This report extends the methods of the previous STR to particular MSW scenarios to try to answer the question more specifically. In addition, a simple diffusion model of neutral current events in the acrylic vessel to describe 'theoretically' the empirical distribution of the MC-generated neutral current events.

Four sets of one year MSW events were generated using SNOMAN 3.00 along with a ^{16}N calibration spectrum. For all scenarios, only events with nhits≥40 were used.

SNOMAN simulation showed that on average there are fewer than 8 nhits per MeV. Forty pmt hits per event implies the trigger is set at about 5 MeV. An acceptance cut of cos-sun≤0.5 was applied to reduce the elastic scattering neutrino. The NC and CC components are then fitted with the MSW-model formula. The charged current neutrino spectrum is obtained by subtracting out the NC spectrum by assuming that it looks like the ^{16}N calibration spectrum normalized to the fitted NC number of events.

The theoretical angular distribution is $\partial\rho/\partial\theta \propto 1-\beta/3*\cos(\theta)$ for charged current events and isotropic for neutral current events (see figs. 1a, 1b). The reduction of the NC and CC events numbers due to the cut to cos-sun≤0.5 is then corrected simply by a factor of 4/3 for the NC and 16/13 (β=1) for the CC events. Due to the angular resolution of the detector (from scattering of the CC-electrons and the reconstruction process), the slope of the angular distribution becomes smaller. Consequently, the correction factor for the CC events is about 5/4 (fitter dependent) instead of 16/13 (see figure 1a, 1b). The difference of the angular distribution between the CC and NC events (and ES as well) is also used for extracting the NC and CC events (see fig. 1c). But due to the angular inaccuracy of the reconstructed neutrino events, the result is not as good as the result using the radial distribution model.

Finally this CC-spectrum was compared with the 'original' CC spectrum of each scenario using goodness-of-fit tests.

The radial distribution model:

Assume the number of neutral current events originating in a small volume in the acrylic vessel is proportional to the local neutron density. This neutron density depends on the source production rate (s), the decay (absorption) rate ($1/\tau$) and diffusion (D). In steady state, the density is constant:

$$\frac{\partial\rho}{\partial t} = s - \frac{\rho}{\tau} + D\nabla^2\rho = 0$$

or

$$\nabla^2\rho - K^2\rho = -\frac{s}{D}$$

with $K^2=1/(D\tau)$. Solving the equation:

$$\rho_{D2O}(r) = C\frac{\sinh(Kr)}{r} + s\tau$$

Outside the vessel, where there is no source, the neutron density is:

$$\rho_{H2O}(r) = A\frac{\exp(-kr)}{r}$$

where $k^2=1/D\tau$ for neutrons in H_2O.
Using the density and the diffusion current boundary conditions $\rho_{D2O}(R=600cm)= \rho_{H2O}(R)$ and $D_{D2O}\nabla\rho_{D2O}(R) = D_{H2O}\nabla\rho_{H2O}(R)$, we obtain:

$$\rho_{D2O}(r) = s\tau(1-\frac{\sinh(Kr)}{r}/(\frac{\sinh(KR)}{R}+B))$$

where $B = \frac{D_{D2O}}{D_{H2O}}(\frac{K\cosh(KR)-\sinh(KR)/R}{kR+1})$. B would be zero if the boundary condition $\rho_{D2O}(R=600cm) = 0$ is applied.

In D_2O with 0.1% H_2O, the thermal diffusion length of a neutron L=1/K is about 120cm. We have taken the neutron parameters from Earle and Wong [1]. Figure 2a shows the Monte-Carlo simulation and the best fit curve. Due to the position uncertainty of the reconstructed events by the time fitter used, which is the source of the CC-radial distribution seen in fig. 2a, the best-fitted curve corresponded to pure D_2O rather than 99.91% enriched. After correcting for position resolution by dividing the NC events by the CC events as a function of position, the Monte-Carlo simulation agrees very well with the theoretically calculated curves as shown in the fig. 2b.

Results:

Radial distribution method:

For all scenarios, a one year spectrum was simulated with the default MSW setting in SNOMAN. NC, CC and ES events were generated separately. These three spectra were then merged to form a 'real' one year spectrum. Then all cutting, fitting and subtracting processes were done on this 'real' spectrum. Firstly, the number of NC and CC events above 5 MeV was determined by fitting the volume distribution to the theoretical shapes given in fig. 1a. Then the NC-energy spectrum was subtracted out by using the ^{16}N γ-spectrum, leaving the CC-spectrum. The tables below give the results for four different scenarios, and the fitted and "true" CC-energy spectra are shown in fig. 3a-d.

1. MSW with large mixing angle ($\Delta m^2=6.2x10^{-6}$, $\sin^2(2\theta)=0.6$):

	cos_sun ≤ 0.5	original (or corrected)	NC/CC
CC "true"	1699	2090	"True"
CC fitted	1736 ± 205	2170 ± 256	0.513
NC "true"	808	1072	Fit
NC fitted	802 ± 178	1069 ± 237	0.493 ± 0.124

2. MSW with small mixing angle ($\Delta m^2=6.2x10^{-6}$, $\sin^2(2\theta)=6.8x10^{-3}$):

	cos_sun ≤ 0.5	original (or corrected)	NC/CC
CC "true"	3124	3911	"True"
CC fitted	3273 ± 179	4091 ± 224	0.274
NC "true"	820	1071	Fit
NC fitted	703 ± 158	937 ± 211	0.229 ± 0.053

3. Vacuum oscillation (Just so) ($\Delta m^2=5.0\text{x}10^{-11}$, $\sin^2(2\theta)=1.0$):

	cos_sun ≤ 0.5	original (or corrected)	NC/CC
CC "true"	2869	3538	"True"
CC fitted	2687 ± 179	3358 ± 245	0.300
NC "true"	807	1056	Fit
NC fitted	1056 ± 160	1408 ± 231	0.419 ± 0.069

4. Sterile neutrino (1/3 of solar neutrino flux only):

	cos_sun ≤ 0.5	original (or corrected)	NC/CC
CC "true"	2808	3530	"True"
CC fitted	2772 ± 136	3460 ± 170	0.096
NC "true"	248	339	Fit
NC fitted	291 ± 117	388 ± 156	0.112 ± 0.045

Angular distribution method:

All operations are similar to the radial distribution method but the results are not as good. The errors are also larger. This could be due to the large angular uncertainty of the reconstruction of the CC events.

1. MSW with large mixing angle ($\Delta m^2=6.2\text{x}10^{-6}$, $\sin^2(2\theta)=0.6$):

	cos_sun ≤ 0.47	original (or corrected)	NC/CC
CC "true"	1699	2090	"True"
CC fitted	1556 ± 433	1960 ± 546	0.513
NC "true"	808	1072	Fit
NC fitted	790 ± 473	1077 ± 645	0.549 ± 0.363

2. MSW with small mixing angle ($\Delta m^2=6.2\text{x}10^{-6}$, $\sin^2(2\theta)=6.8\text{x}10^{-3}$):

	cos_sun ≤ 0.5	original (or corrected)	NC/CC
CC "true"	3124	3911	"True"
CC fitted	2721 ± 460	3428 ± 580	0.274

NC “true”	820	1071	Fit
NC fitted	940 ± 502	1282 ± 685	0.373 ± 0.209

3. Vacuum oscillation (Just so) (Δm^2=5.0x10^{-11}, $\sin^2(2\theta)$=1.0):

	cos_sun ≤ 0.5	original (or corrected)	NC/CC
CC “true”	2869	3538	“True”
CC fitted	3088 ± 485	3890 ± 611	0.300
NC “true”	807	1056	Fit
NC fitted	279 ± 530	380 ± 723	0.098 ± 0.136

4. Sterile neutrino (1/3 of solar neutrino flux only):

	cos_sun ≤ 0.5	original (or corrected)	NC/CC
CC “true”	2808	3530	“True”
CC fitted	2482 ± 494	3127 ± 622	0.096
NC “true”	248	339	Fit
NC fitted	332 ± 540	452 ± 736	0.145 ± 0.238

Goodness-of-fit tests:

The χ^2 test and the Kolmogorov-Smirnov test are two commonly used goodness-of-fit testing methods. In the χ^2 test, two discrete distributions *F(x)* and *S(x)* are compared and the value

$$\chi^2 = \sum_{i=1}^{n} \frac{(F(x_i) - S(x_i))^2}{F(x_i) + S(x_i)}$$

is calculated. For a value of χ^2 and a degree of freedom of ν, the χ^2 has a distribution function of:

$$P(\nu, \chi^2) = \frac{\int_0^{\chi^2} e^{-t} t^{\frac{\nu}{2}-1} dt}{\int_0^{\infty} e^{-t} t^{\frac{\nu}{2}-1} dt}$$

To accept the hypothesis that the two distributions are the same, P=0.5 is usually chosen. To reject the hypothesis, the level of significance is often set to >0.9.

The Kolmogorov-Smirnov test compares two discrete event distribution sets X_i and Y_i by generating two functions so that $F(x_i) = i/n$ for the set X between x_i and x_{i+1} and $S(y_i) = i/n$ for the set Y. A value D is calculated as D = max($F(x_i)$ - $S(y_i)$). The distribution function of D

$$P(\sqrt{n}D \leq t) = 1 - 2\sum_{i=1}^{\infty}(-1)^{i-1}e^{-2i^2t^2}$$

can be used to accept or reject the hypothesis whether $X\ and\ Y$ have the same distribution. For the neutrino spectrum, the data set X or Y are not fully discrete because there is more than one event in the same nhit bin. Instead they were divided uniformly across the bin. Fig. 3e-3h show the cumulative distribution for the "true" and fitted CC spectra.

Discussion:

The results of extracting NC/CC and obtaining the CC shape from SNO 'raw' data are fairly good using the radial distribution of events except for the "just so" vacuum oscillation scenario which fitted more NC events than there should be. However, the CC shape of the "just so" scenario is quite different from the other scenarios' and might allow it to be distinguished. The goodness-of-fit tests for all "true" CC and extracted CC distribution in all scenarios passed the acceptance criteria except for the χ^2 test for the "just so" CC distribution.

Acknowledgment:

We are grateful to B. G. Nickel of University of Guelph for useful discussions and help in the neutron diffusion model.

References:

1. E.D. Earle and P. Y. Wong, 1987, Collection of Annexes in Support of Main Proposal, SNO-87-12, Annex 1.

2. W. Frati and E. Beier, Extraction of CC NC and ES Signals - III Multi-Year Three Parameter Maximum Likelihood Analysis, SNO-STR-91-025

3. J. J. Simpson and J.-X. Wang, Determination of NC/CC Ratio During the Pure D_2O Stage of Operation, SNO-STR-98-001

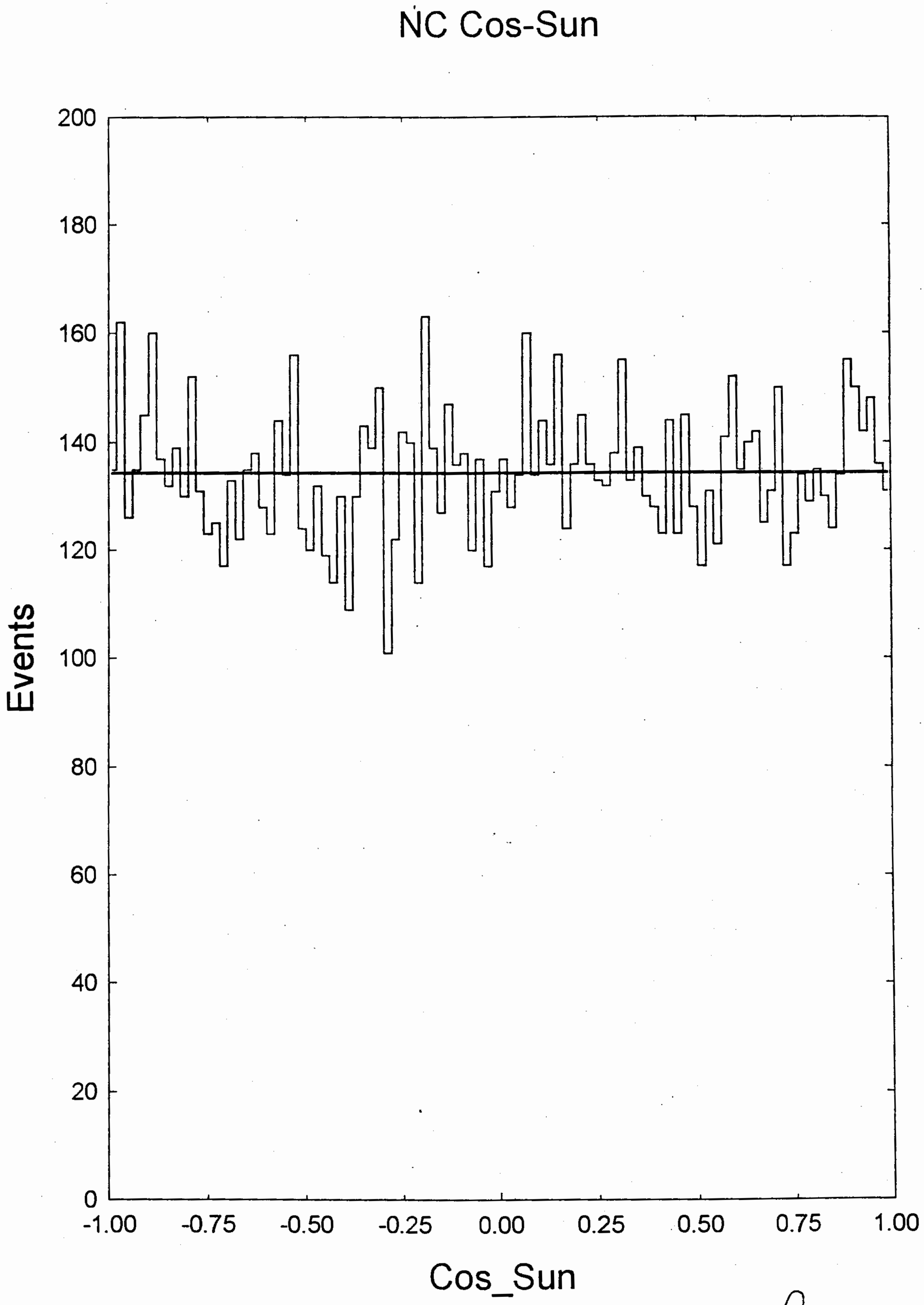
NC Cos-Sun
Events
Cos_Sun
0
20
40
60
80
100
120
140
160
180
200
-1.00
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.00

fig 1a

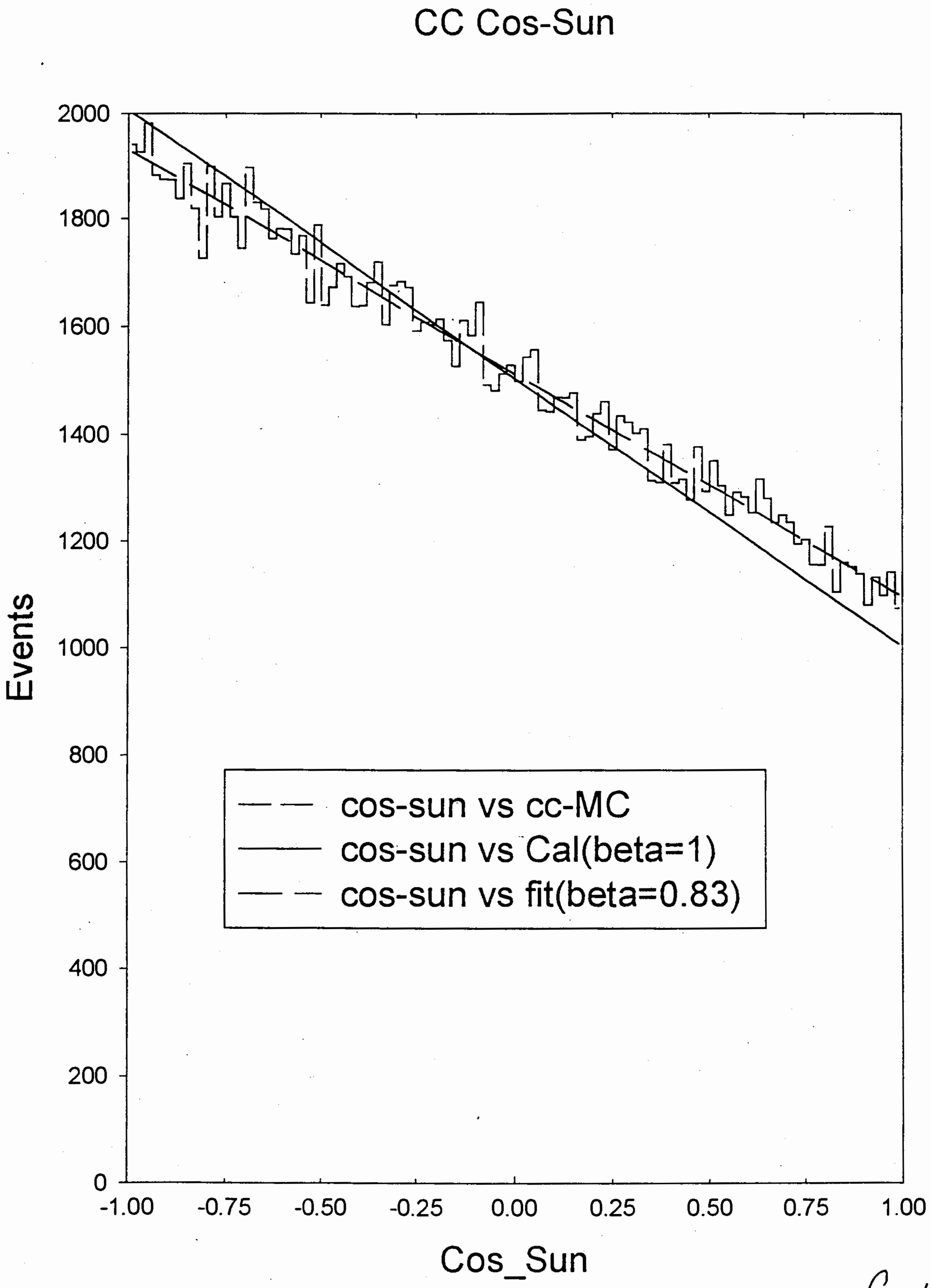
CC Cos-Sun
Events
2000
1800
1600
1400
1200
1000
800
600
400
200
0
-1.00
-0.75
-0.50
-0.25
0.00
0.25
0.50
0.75
1.00
Cos_Sun
cos-sun vs cc-MC
cos-sun vs Cal(beta=1)
cos-sun vs fit(beta=0.83)

fig. 1b

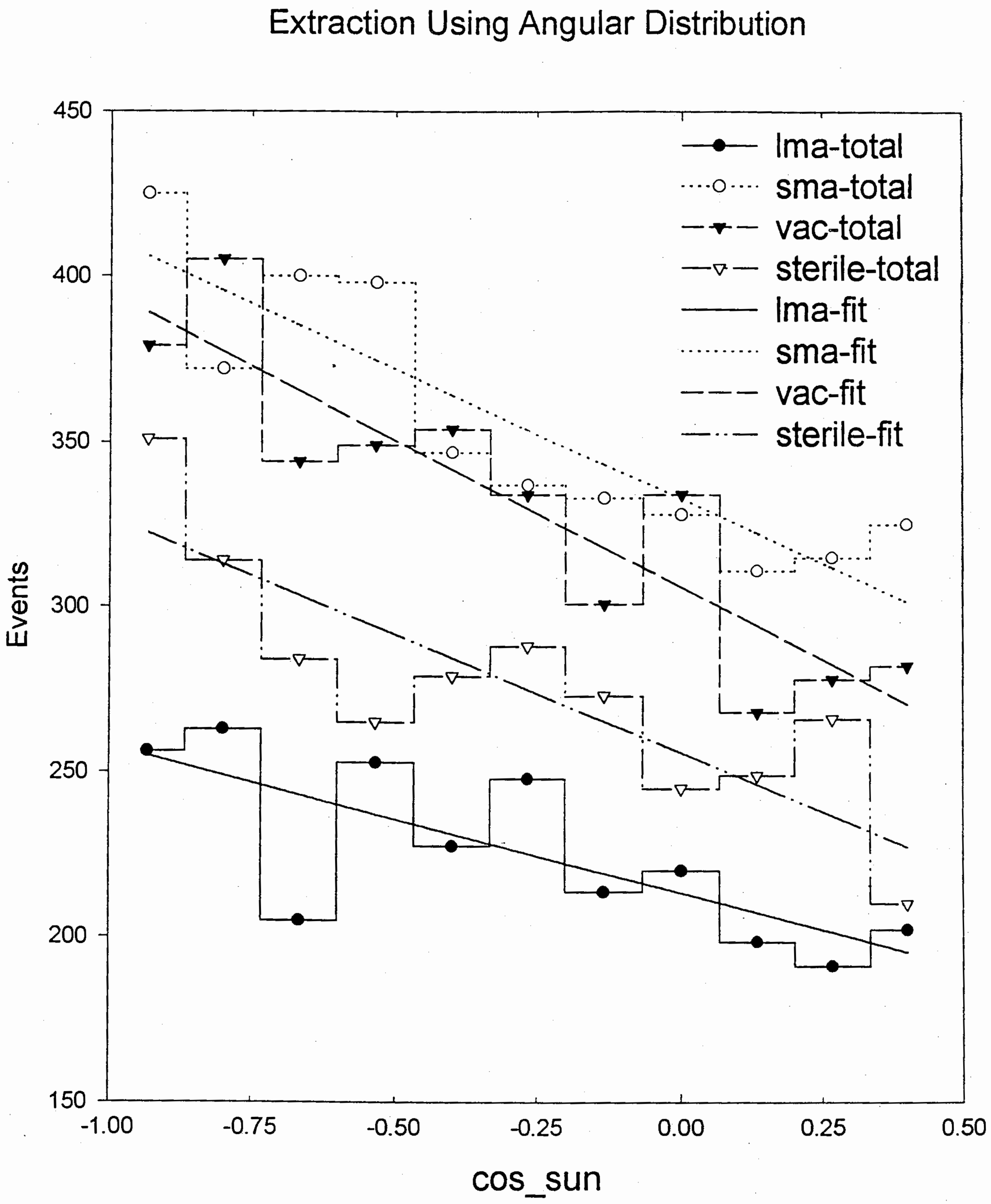
Extraction Using Angular Distribution
450
400
350
300
250
200
150
Events
-1.00
-0.75
-0.50
-0.25
0.00
0.25
0.50
cos_sun
lma-total
sma-total
vac-total
sterile-total
lma-fit
sma-fit
vac-fit
sterile-fit

fig. 1c

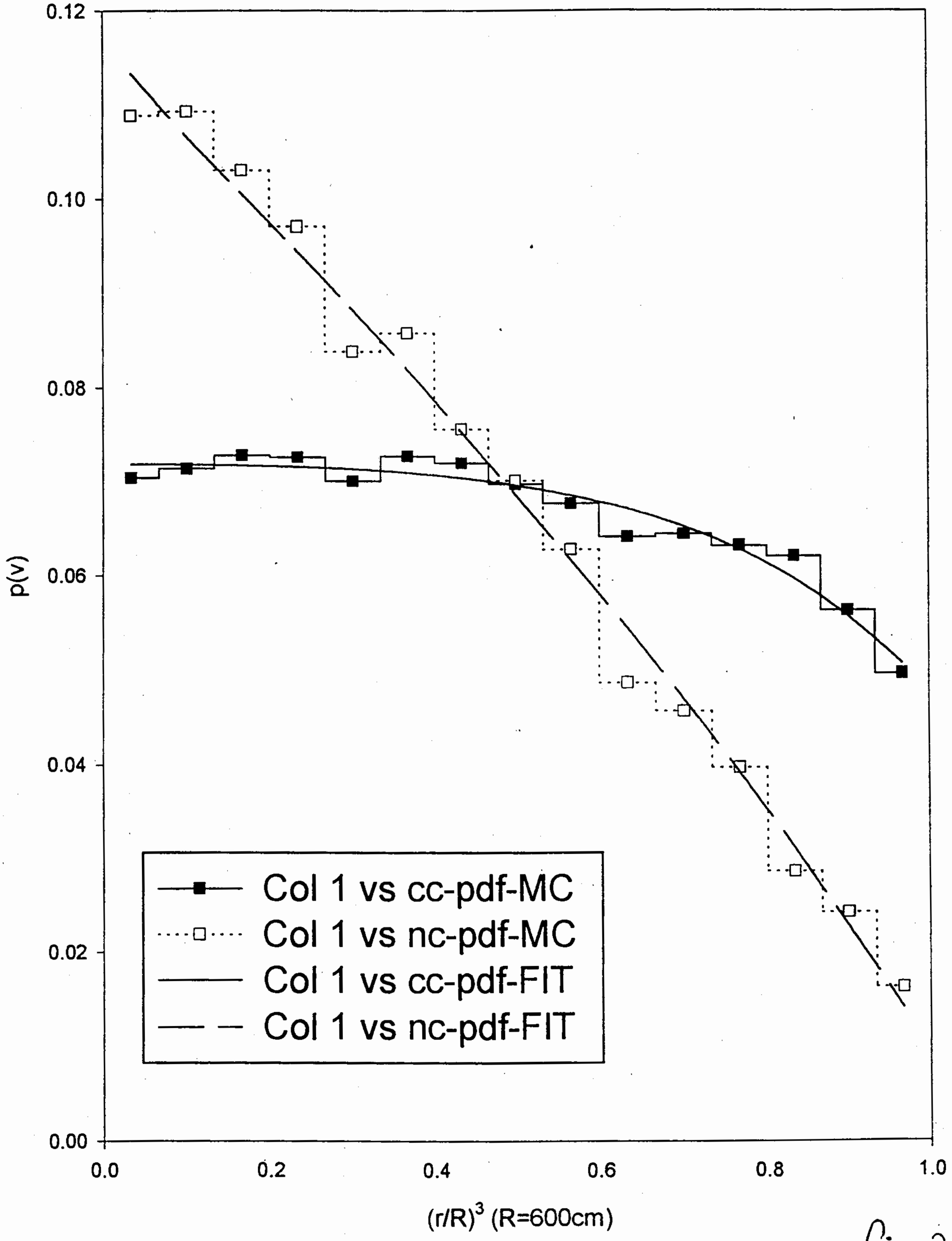
0.12
0.10
0.08
0.06
0.04
0.02
0.00
p(v)
0.0
0.2
0.4
0.6
0.8
1.0
$(r/R)^3$ (R=600cm)
Col 1 vs cc-pdf-MC
Col 1 vs nc-pdf-MC
Col 1 vs cc-pdf-FIT
Col 1 vs nc-pdf-FIT

fig. 2a

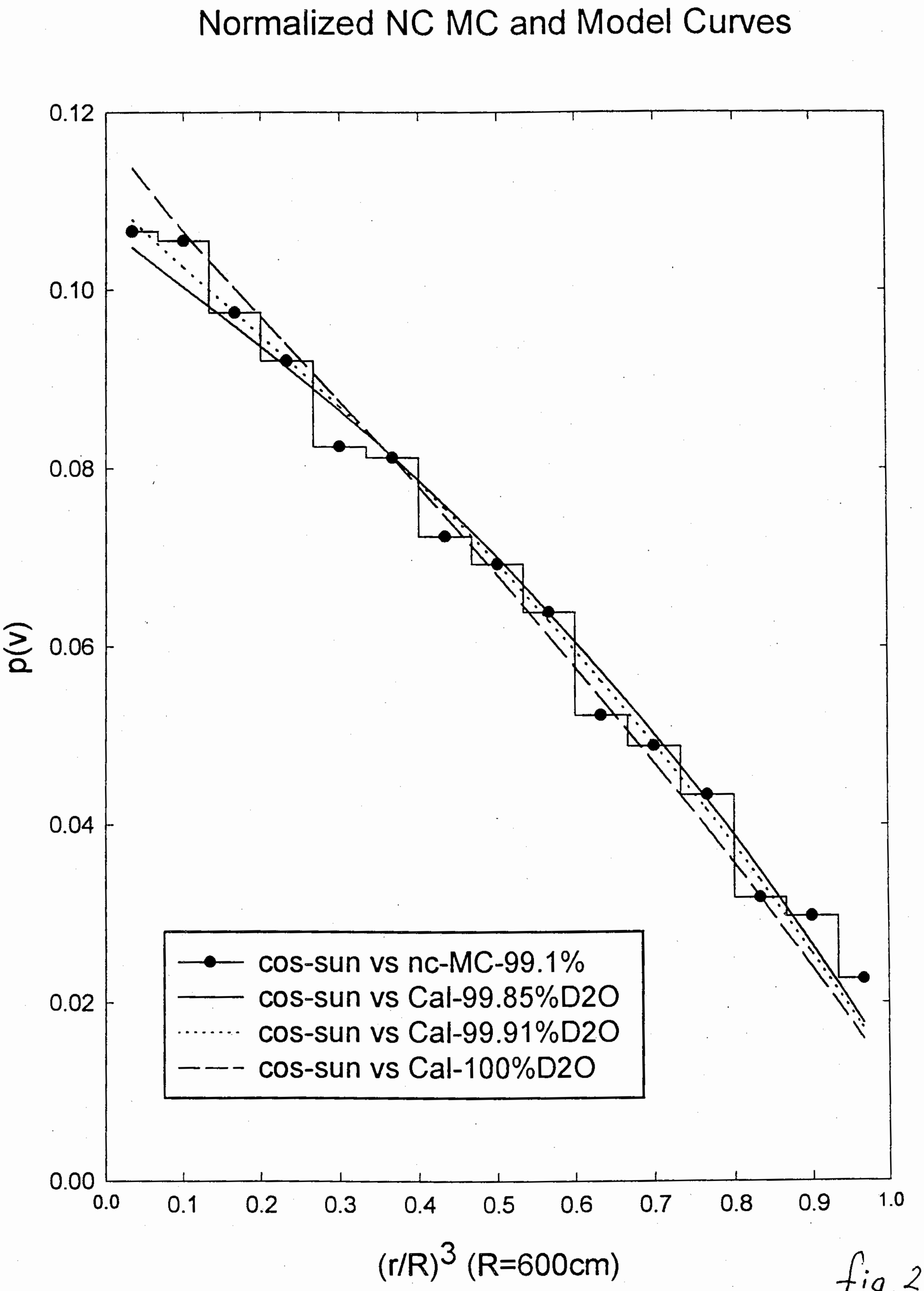
Normalized NC MC and Model Curves
p(v)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
(r/R)^3 (R=600cm)
cos-sun vs nc-MC-99.1%
cos-sun vs Cal-99.85%D2O
cos-sun vs Cal-99.91%D2O
cos-sun vs Cal-100%D2O

fig. 2b

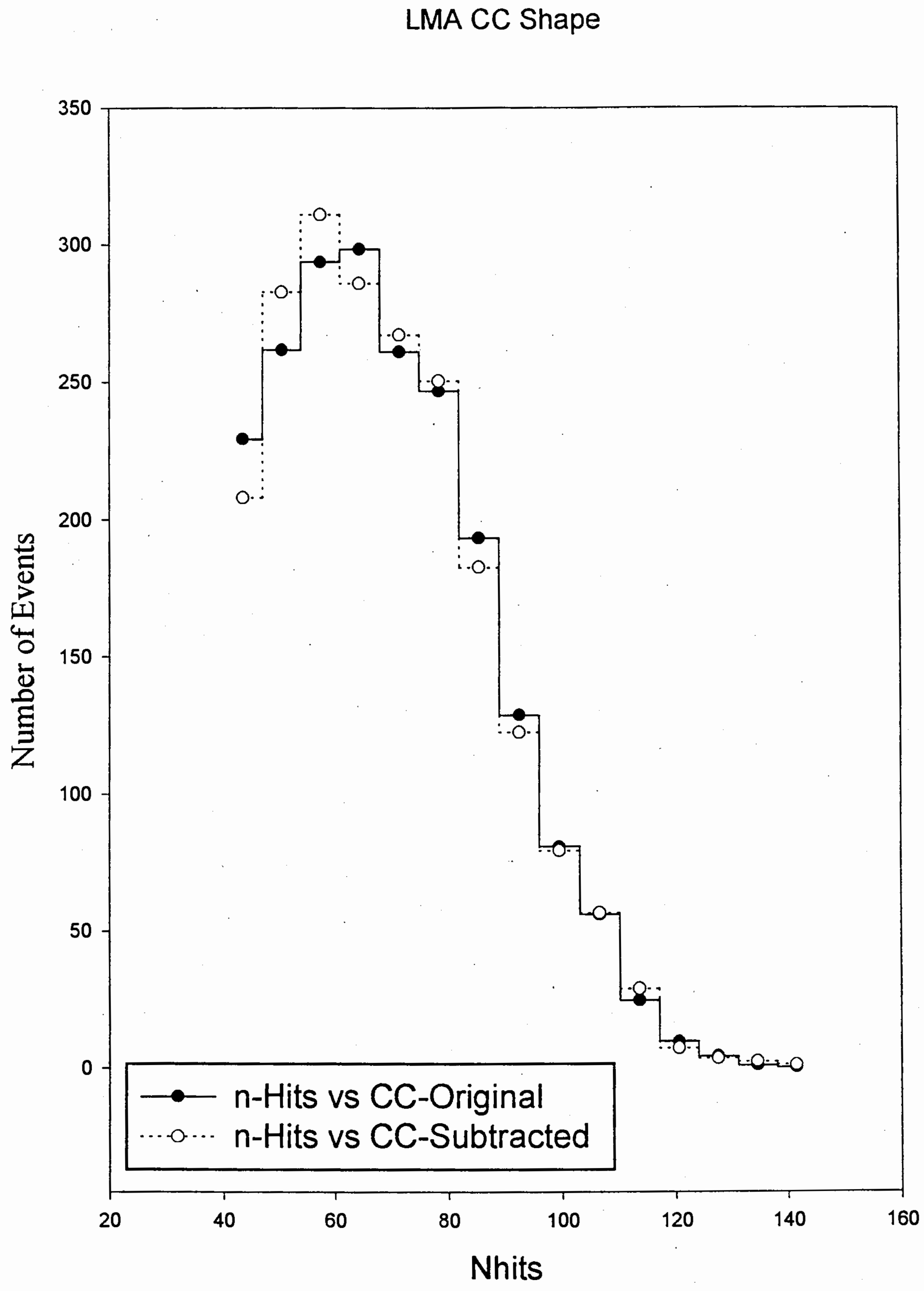
LMA CC Shape
Number of Events
350
300
250
200
150
100
50
0
20
40
60
80
100
120
140
160
Nhits
n-Hits vs CC-Original
n-Hits vs CC-Subtracted

fig.3a

SMA CC Shape

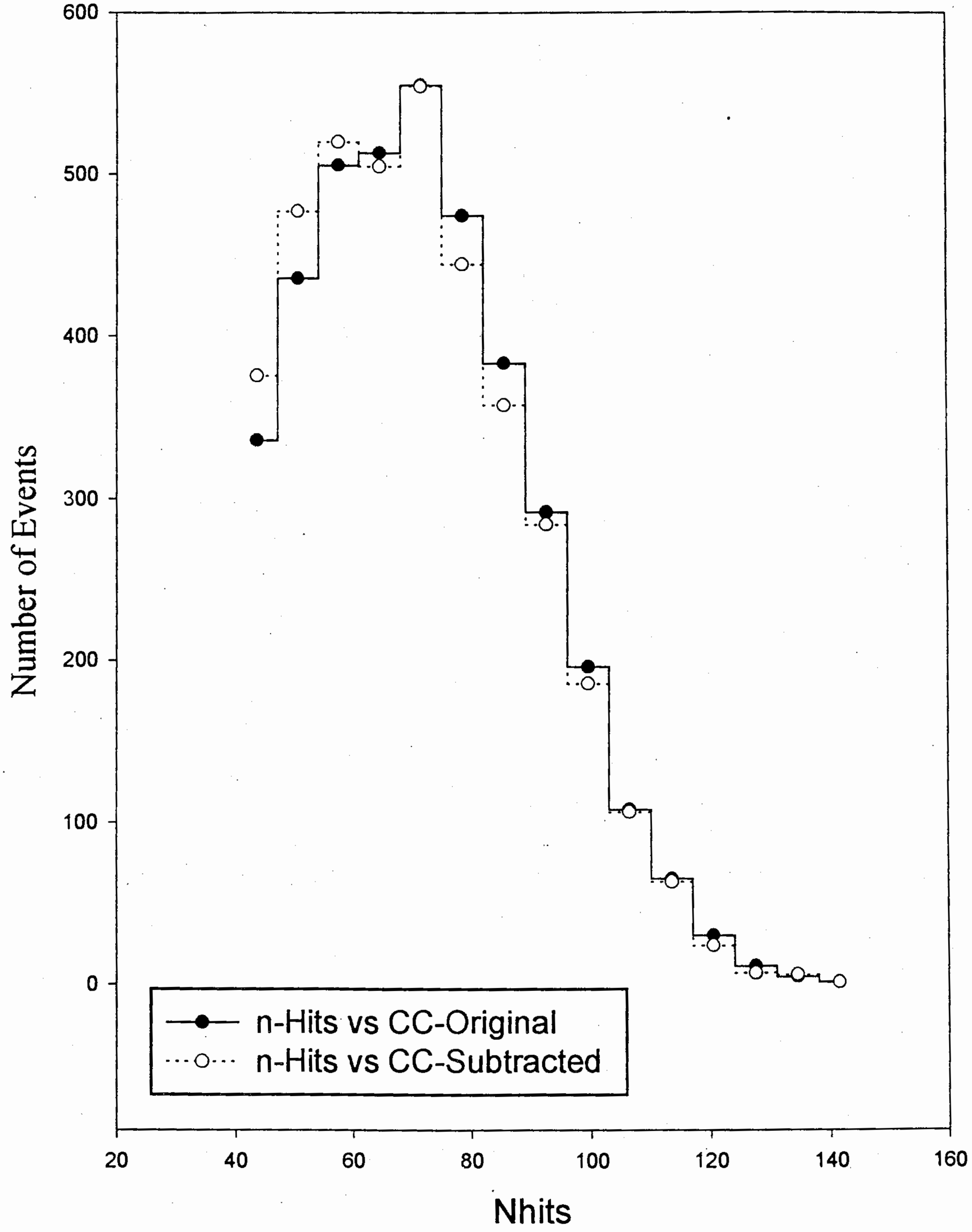

fig. 3b

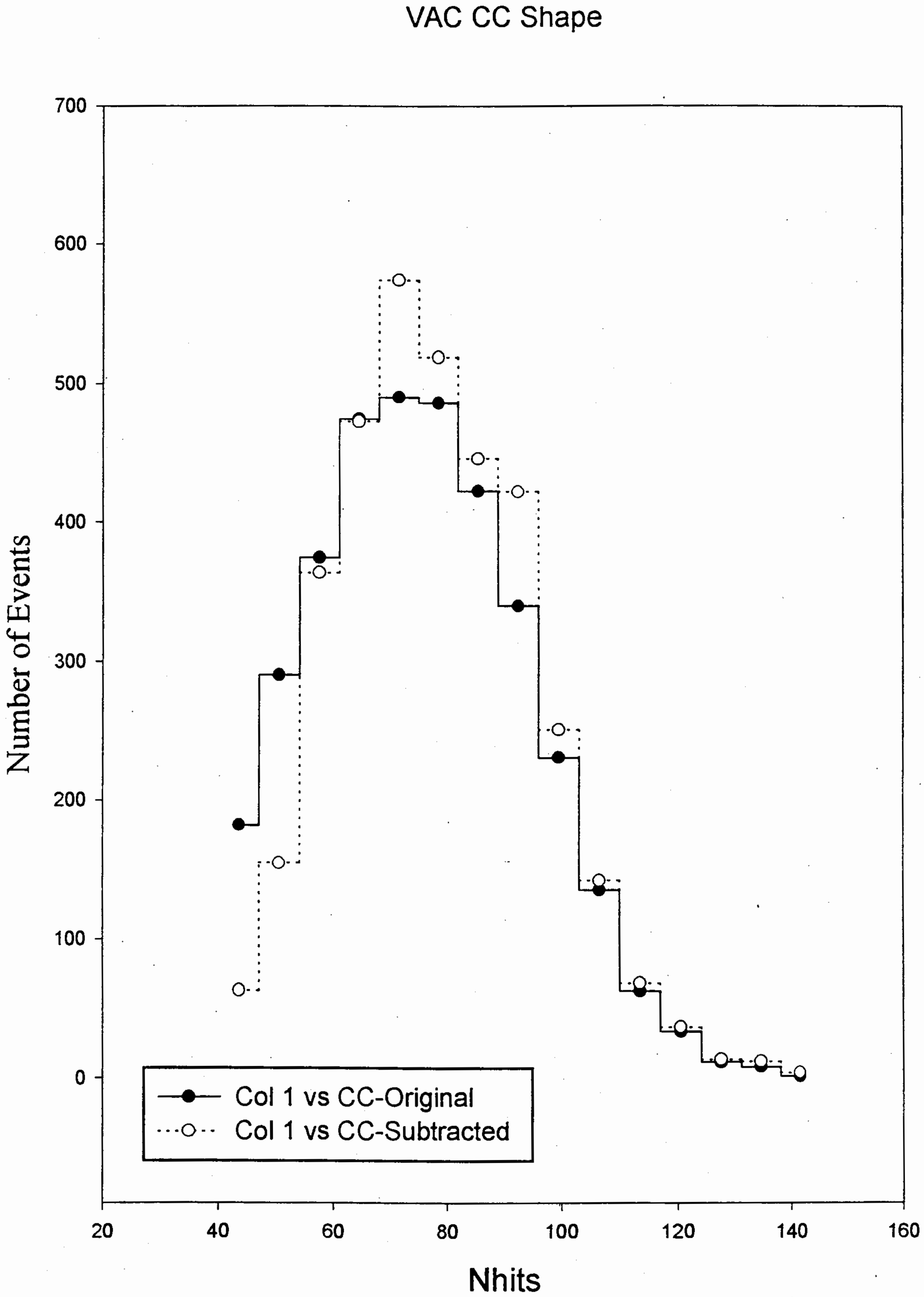
VAC CC Shape
Number of Events
700
600
500
400
300
200
100
0
Nhits
20
40
60
80
100
120
140
160
Col 1 vs CC-Original
Col 1 vs CC-Subtracted

fig. 3c

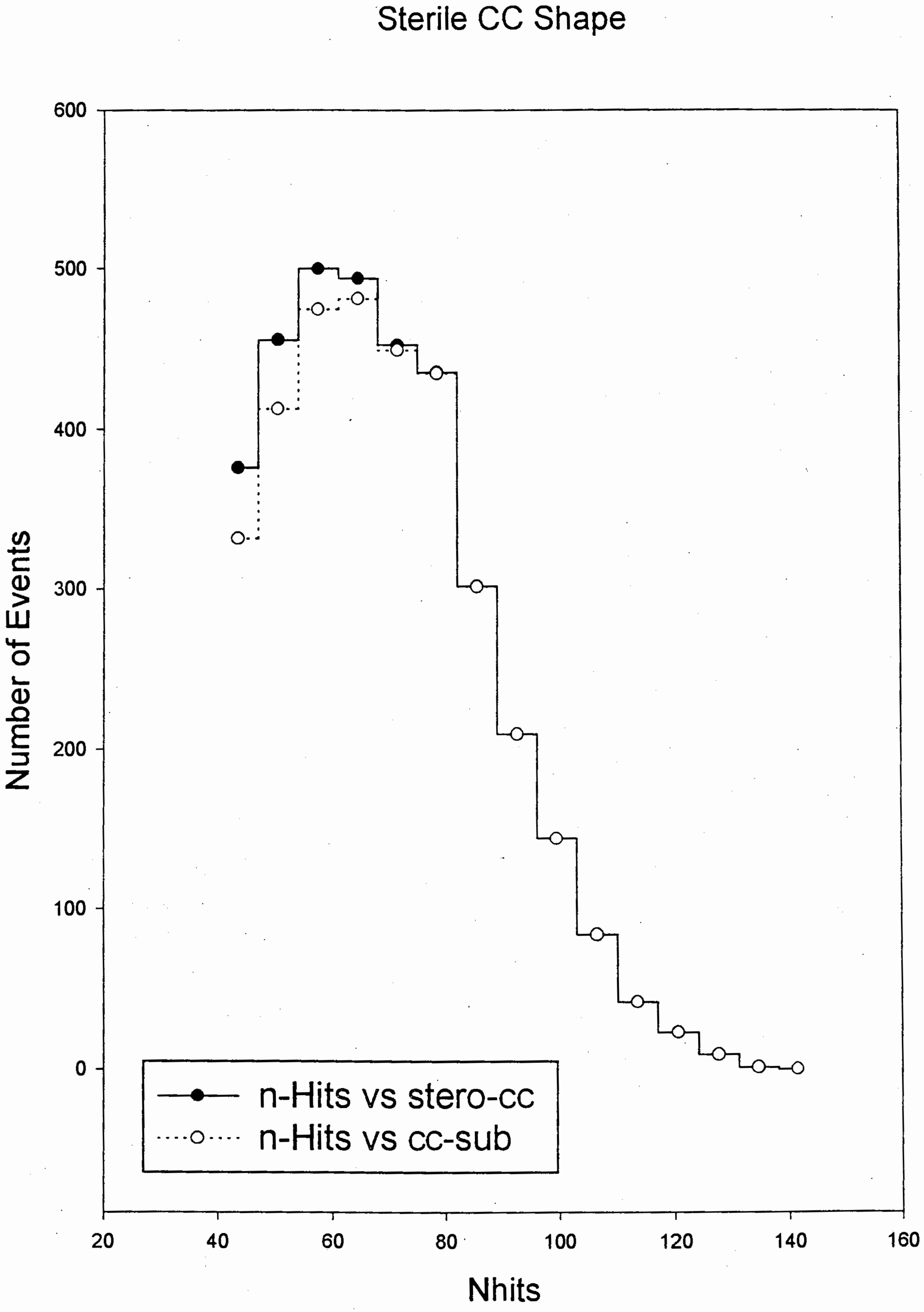
Sterile CC Shape
Number of Events
600
500
400
300
200
100
0
20
40
60
80
100
120
140
160
Nhits
n-Hits vs stero-cc
n-Hits vs cc-sub

fig. 3d

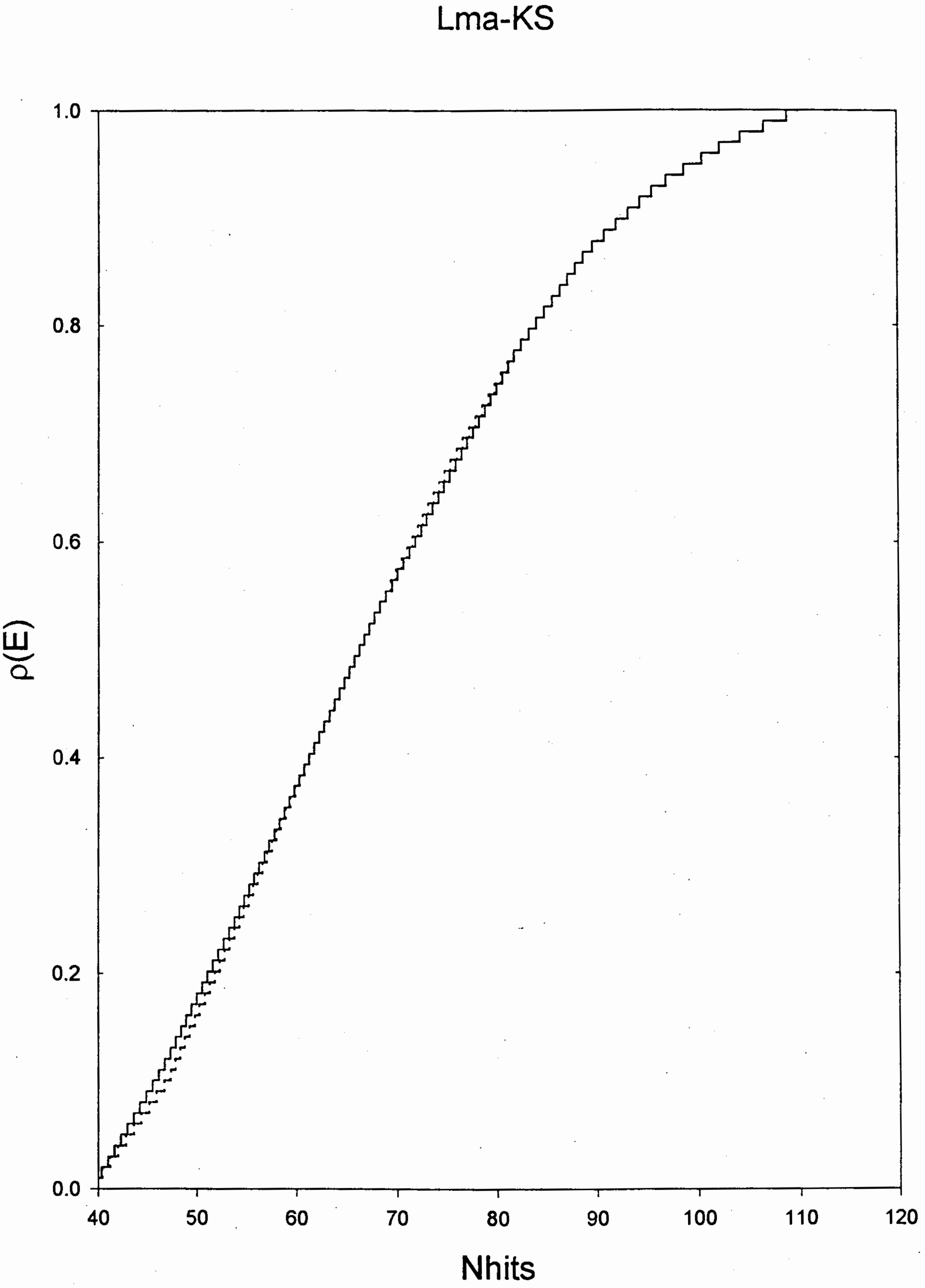

fig. 3e

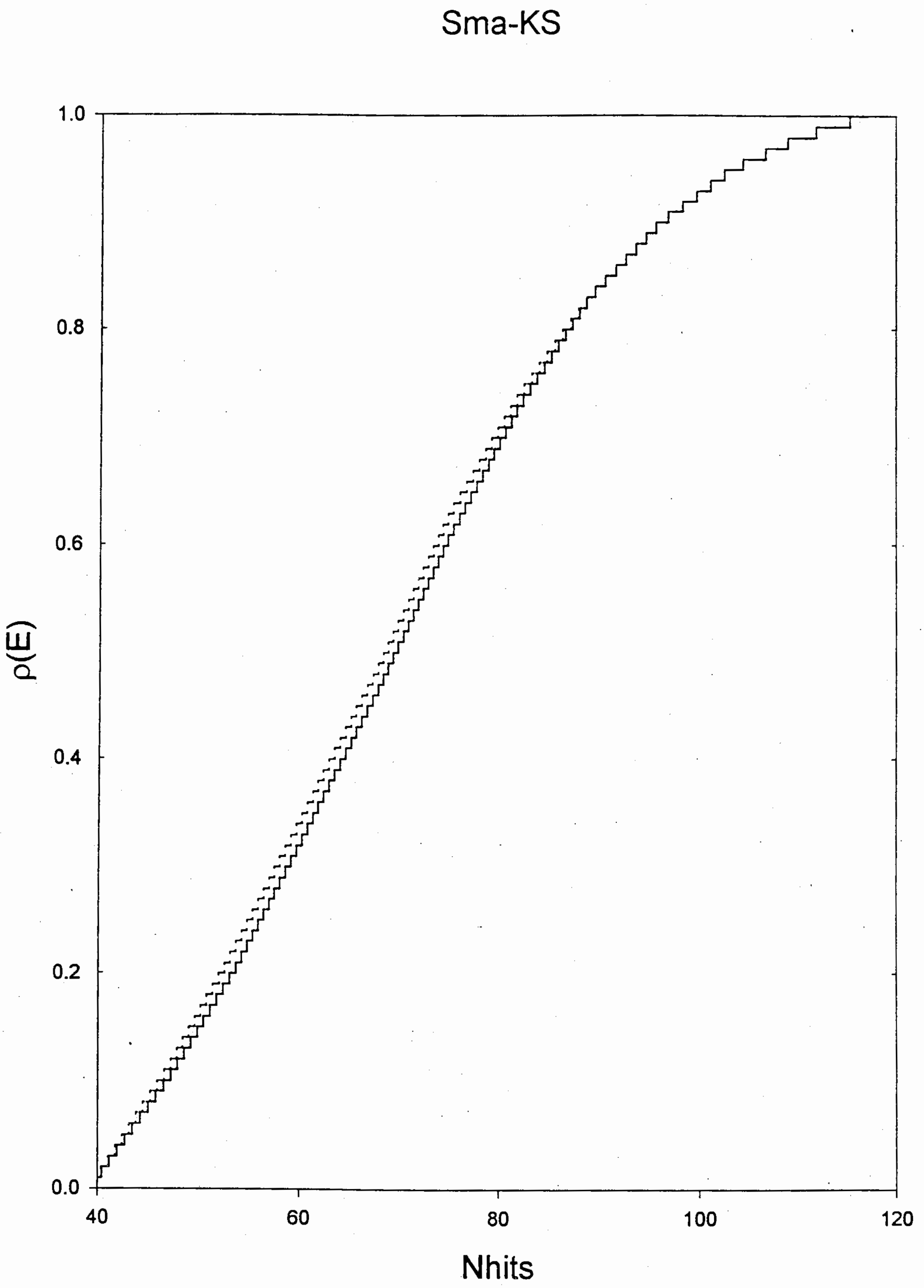

fig.3f

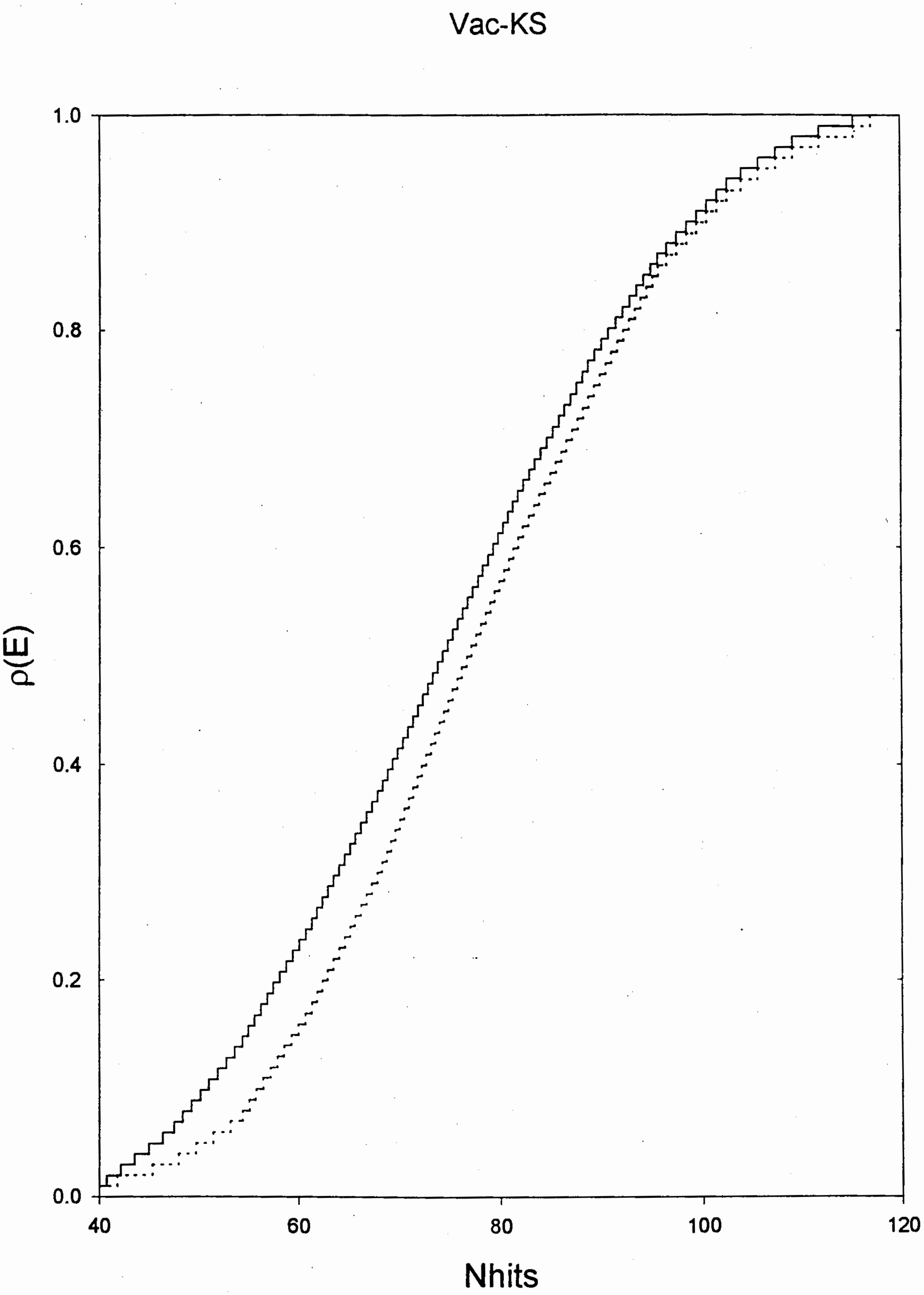

fig. 3g

10

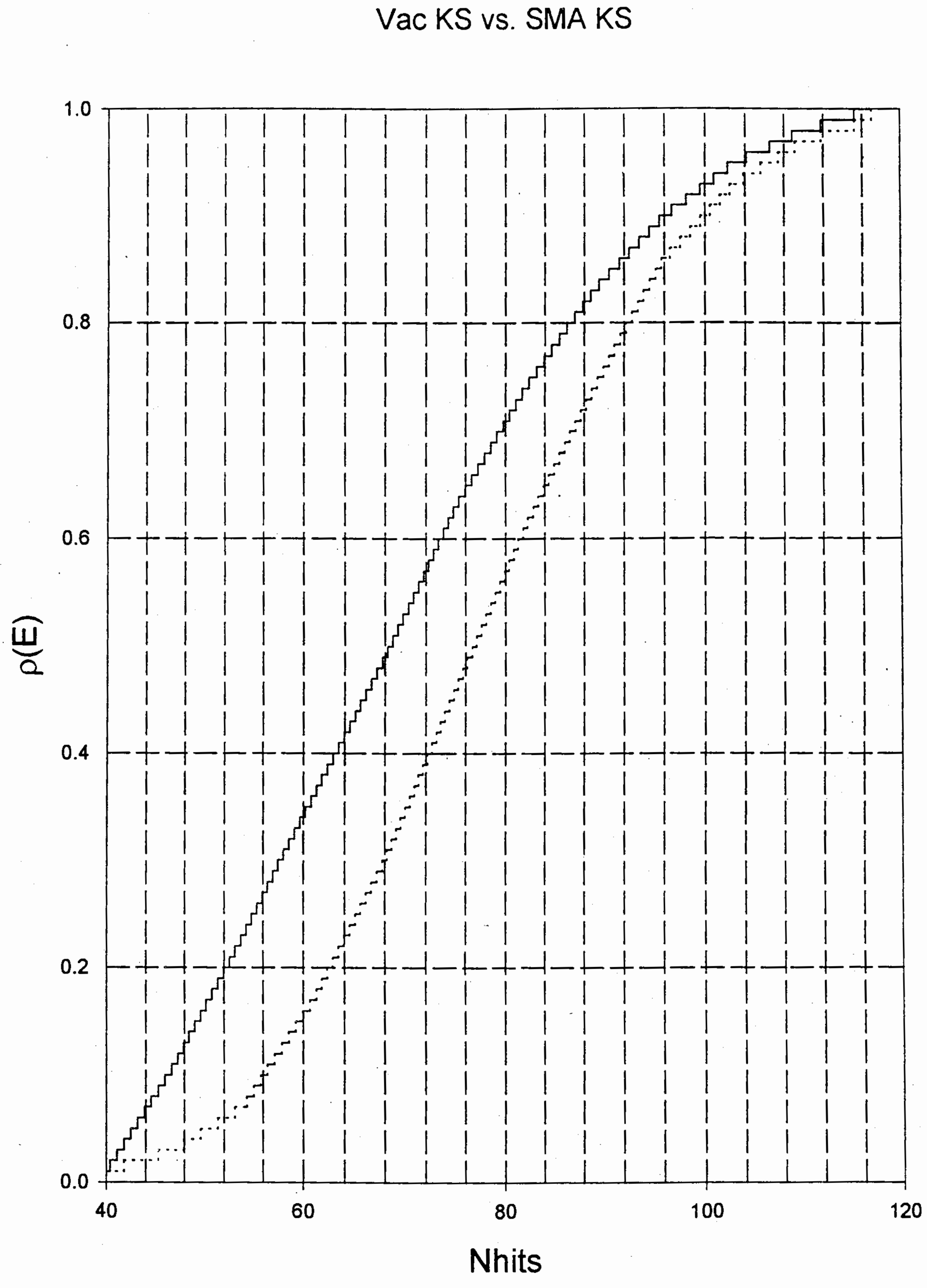

fig. 3h