Internal bremsstrahlung and SNO

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SNO–STR–97–021

1. Introduction

The best calculations that money can buy for the spectrum of electrons emitted by ^{8}Li or in the process $\nu_e + d \to ppe^-$ include radiative corrections. The real photons involved constitute the internal bremsstrahlung and in SNO they will not escape detection: electron spectra in SNO will be shifted up a trifle from those of the best calculations that money can buy. Since internal bremsstrahlung — radiation inevitably accompanying the sudden appearance of an electron walloping along — is O(α), effects will be at the 1% level. Here I estimate these effects rather more accurately.

2. Internal bremsstrahlung spectrum

I estimated the number of photons radiated,as a function of energy, from the semi-classical treatment to be found in [1].

$$\frac{dN_\gamma}{dE_\gamma} = \frac{e^2}{\pi}\frac{1}{E_\gamma}\left\{\frac{1}{\beta}\,ln\,\frac{1+\beta}{1-\beta} - 2\right\} \simeq \frac{e^2}{\pi}\frac{1}{E_\gamma}2\left\{ln\;2\gamma - 1\right\} \qquad (1)$$

for $\beta \sim 1$. This I approximated as

$$dN_\gamma = 10^{-2}\frac{dE_\gamma}{E_\gamma} \qquad (2)$$

The number of photons radiated diverges as $E_\gamma \to 0$ but this is handled by imposing a (very) low energy cutoff which does not appear in the electron spectrum.

3. Effect of internal bremsstrahlung on electron spectra.

Set

$$\frac{dN_\gamma}{dE_\gamma} = \frac{K}{E_\gamma}$$

Then the number of electrons which do not radiate is $1 - Kln\,\frac{E_{\gamma\,\mathrm{max}}}{E_{\gamma\,\mathrm{min}}}$ where $E_{\gamma\,\mathrm{max}}$ is the electron energy without radiation and $E_{\gamma\,\mathrm{min}}$ is the low energy cutoff. If the spectrum without radiation is $f(E)$ and the electron energy after radiation is E', then these electrons contribute

$$\left(1 - K\;ln\frac{E_{\gamma\,\mathrm{max}}}{E_{\gamma\,\mathrm{min}}}\right) f(E') \qquad (3)$$

with $E' = E$.

For those which DO radiate, $E = E' + E_\gamma$ and the spectrum generated is

$$K \int_{E_{\gamma\,\min}}^{E_0 - E'} f(E' + E_\gamma) \frac{dE_\gamma}{E_\gamma} \tag{4}$$

where E_0 is the end point of the spectrum. The spectrum of detected energy E', assuming that no internal bremsstrahlung is detected, is the sum of (3) and (4). This can be written as

$$f(E')(1 + \delta(E', E_0))$$

I took the approximate form

$$f(E') = E^2(E_0 - E)^2 \tag{5}$$

in order to find the factor $\delta(E', E_0)$. It is obvious that, on setting $E = E' + E_\gamma$ and expanding, that one term in the sum will be $f(E')$. When this term is integrated in (4) it yields a logarithmic term which cancels $E_{\gamma\,\min}$ in (3). The result is

$$\delta = K \left\{ ln(x-1) + \frac{1}{12}(x^2 + 6x - 25) \right\}$$

where $x = \frac{E_0}{E'}$

Taking E_0 to be 15 MeV yields

Table 1

E' (MeV)	$\delta(E')$
2	+0.081
4	+0.020
6	+0.001
8	-0.010
10	-0.018
12	-0.027
14	-0.041

The largest corrections are at the ends of the spectrum — and there isn't much there. Across the middle of the spectrum

$$\frac{d\delta(E')}{dE'} \simeq -0.005 \text{ MeV}^{-1}$$

(This matches well the slope of the radiative correction factor calculated from QED [2].)

The peak shift resulting from the modification

$$f(E) \rightarrow f(E')[1 - bE']$$

is

$$-\frac{bE_p^2}{4}$$

where E_p is the peak of the original spectrum, $E_p = \frac{1}{2}E_0$. This shift is -0.07 MeV. If the mean is evaluated over the range 4–11 MeV, the shift is -0.065 b E_p^2 which is -0.02 MeV. Thus if a theoretical form including radiative corrections were to be compared with data in which ALL internal bremsstrahlung energy were recovered the data would differ from the model by a factor $1 + 5 \times 10^{-3}E'$, the peak in the data would lie 0.07 MeV higher than the theoretical peak and the 4-11 MeV mean energy would lie 0.02 MeV higher. I doubt that SNO data will be sensitive to such effects. In fact, ALL the bremsstrahlung energy will NOT be recovered and realistically the effects are half those evaluated under the assumption of complete recovery.

4. How much energy is recovered?

Internal bremsstrahlung will feed light into SNO by Compton scattering (and occasionally by pair production). The light yield per MeV of electron energy is zero below the Cerenkov threshold, 0.16 of the asymptotic value at 0.5 MeV and half the asymptotic value at 1 MeV. Multiple Compton scatters degrade the photon energy in a few bounces and once the photon has been degraded below 1 MeV the next Compton electron will contribute negligibly to the light output. A simple Monte Carlo calculation showed that the light output corresponds to the initial photon energy LESS $\sim$ 1 MeV, almost independent of the energy of the initial photon.

I therefore assumed that the energy of any radiated photon $<$ 1 MeV is lost and that the energy of any radiated photon above 1 MeV is recovered LESS 1 MeV. The spectrum of detected energy E' is then made up from three pieces

$$f(E')\left[1 - K\ ln\ \frac{E'}{E_{\gamma\,\min}}\right] \tag{6}$$

This is the contribution from electrons which do not radiate, as in (3).

$$K\int_{E_{\gamma\,\min}}^{\varepsilon} f(E'+E_\gamma)\frac{dE_\gamma}{E_\gamma} \tag{7}$$

This is the contribution of electrons which radiate between $E_{\gamma\,\min}$ and ε, for which the radiated energy is lost as in (4).

$$K\int_{\varepsilon}^{E'+\varepsilon} f(E'+\varepsilon)\frac{dE_\gamma}{E_\gamma}(E'+\varepsilon \le E_0) \tag{8}$$

This is the contribution of electrons and radiated photons which together yield detected energy ε below that of the original electron. The cutoff ε is $\sim$ 1 MeV. The integrals are trivial and when the sum of (6) – (8) is written in the form

$$f(E')(1 + \delta_\varepsilon(E', E_0))$$

the numerical results are

Table 2

E' (MeV)	$\delta_\varepsilon(E')$
2	+0.023
4	+0.010
6	+0.004
8	-0.001
10	-0.007
12	-0.016
14	-0.041

(The agreement between the entries for 14 MeV in Tables 1 and 2 is not a coincidence.)

Below 2 MeV $\delta(E')$ diverges rapidly from $\delta_\varepsilon(E')$; in the former case energetic electrons can feed the low energy end of the detected energy spectrum but in the latter it is impossible.

Across the middle of the spectrum

$$\frac{d\delta_\varepsilon(E')}{dE'} \simeq -0.0025 \text{ MeV}^{-1}$$

The detected energy spectrum will differ from the theoretical form with radiative corrections by a factor

$$1 + 2.5 \times 10^{-3} E'.$$

Consequently the peak will lie $\sim$ 0.035 MeV higher than the theoretical peak and the 4–11 MeV mean $\sim$ 0.01 MeV higher.

5. Conclusion

Anyone who thinks that SNO will be sensitive to these tiny effects should worry about them! But theoretical spectra are not reliable at this level.

References

[1] J D Jackson Classical Electrodynamics (p 526,1st ed.)

[2] A Sirlin Phys.Rev. **164** (1967) 1767