SNO-STR — 94-050

Analytical Calculation of the Number of SNO PMT Hits for Mono-energetic Electrons Borne at the Centre of the SNO Active Volume

C.K. Hargrove and F. Dalnoki-Veress
Centre for Research in Particle Physics, Ottawa, Ontario

INTRODUCTION:

A program was written to analytically calculate the number of SNO PMT hits of a mono-energetic electron created at the centre. No attempt has been made to include all the factors influencing the number of PMT hits, rather attention has been given to the dominant effects in order to keep the calculation simple. Of the neglected effects only Bremsstrahlung will increase the number of SNO PMT hits, thus this approximation is valid at low energies. The remainder of the phenomena not included will decrease the total number of PMT hits. Consequently, the results presented can be seen only as an upper bound to the total number of PMT hits.

CALCULATIONAL DETAILS:

The number of SNO PMT hits were calculated using the following prescription:

$$SNOPMTHITS = \int_{E}^{Cer} \frac{(\frac{dN}{dx})}{(\frac{dE}{dx})_{BB} + (\frac{dE}{dx})_{Brem}} dE * Efficiencies$$

E=Energy of electron in MeV
Cer=Cerenkov threshold energy for heavy water

1. Number of SNO PMT hits per unit path:

$$\frac{dN}{dx} = 2\pi\alpha\left[1 - \frac{1}{n^2}\left(\frac{E^2}{E^2 + 0.511^2}\right)\right]\int \gamma(\omega)\, d\omega$$

α=1/137
ω=Frequency of photons in /cm
$\gamma(\omega)$=PMT response as given in SNOMAN
n=Index of refraction

-It is assumed that the index of refraction does not vary significantly over the range of sensitivity of the PMT's. (See Nordling and Osterman, "Physics Handbook", Table 4.3)

-Not taken into account as discussed earlier is the effect of the Bremsstrahlung photons in creating electrons resulting in Cerenkov photons.

-Cerenkov energy loss is not included in the calculation since it is <1% of the sum of Bethe-Bloch and Bremsstrahlung energy loss.

3. Notes on Efficiencies:

-Photocathode coverage efficiency in the calculation is 65%
-Probability of a photon being absorbed is calculated from a separate Monte Carlo program using a mean absorption and scattering path length as suggested in ANNEX 6.
(R.C. Allen et al. A Monte Carlo Simulation of the Sudbury Neutrino Observatory Heavy Water Cerenkov Detector)
This program calculates on an event-by-event basis the absorption probability using the assumption of an isotropic scattering distribution, 40 m absorption and scattering mean free paths for the H_2O, D_2O. A value of 0.8041 obtained for the absorption probability is inserted as an efficiency in the calculation of the total number of PMT hits.

4. Multiple Scattering:

-It is assumed that the effect of multiple scattering is not significant in the program for a few reasons that should be taken together:

a) The medium is homogeneous and so if direction is not important energy loss is the same in any direction.

b) The photon mean absorption length is 40m so the attenuation due to an uncertainty in position (caused by multiple scattering) is negligible for a first order calculation.

5. Optical Effects in the Acrylic:

-The optical effects at the acrylic/D_2O boundary are neglected because of the following reasons:

a) As can be seen from the photon transport Monte Carlo, the effect of scattering on the absorption probability is very small. Consequently, as an approximation the majority of photons can be considered to travel straight out from the centre incident at the acrylic/D_2O interface along the normal. Thus the effects of refraction and reflection are neglected as a first order approximation.

-Integral over the response curve of the sensitivity of the PMT's convolved with the transmittance of the acrylic was found to be 2479 /cm in a separate program.
-Not taken into account is PMT noise since it can be turned off in SNOMAN for comparison purposes.

2. Energy loss:

a) Bethe-Bloch Energy loss:

$$-\left(\frac{dE}{dx}\right)_{\mu} = 0.1535\rho\frac{Z}{A}\left(\frac{E^2}{E^2+0.511^2}\right)\left[\ln\left(\frac{k^2(k+2)}{2(I/0.511)^2}\right)+1-\left(\frac{E^2-0.511^2}{E^2}\right)+\frac{(k^2/8-(2k+1)\ln 2)}{(k+1)^2}\right.$$

$$\left.-\delta-2*\frac{C}{Z}\right]$$

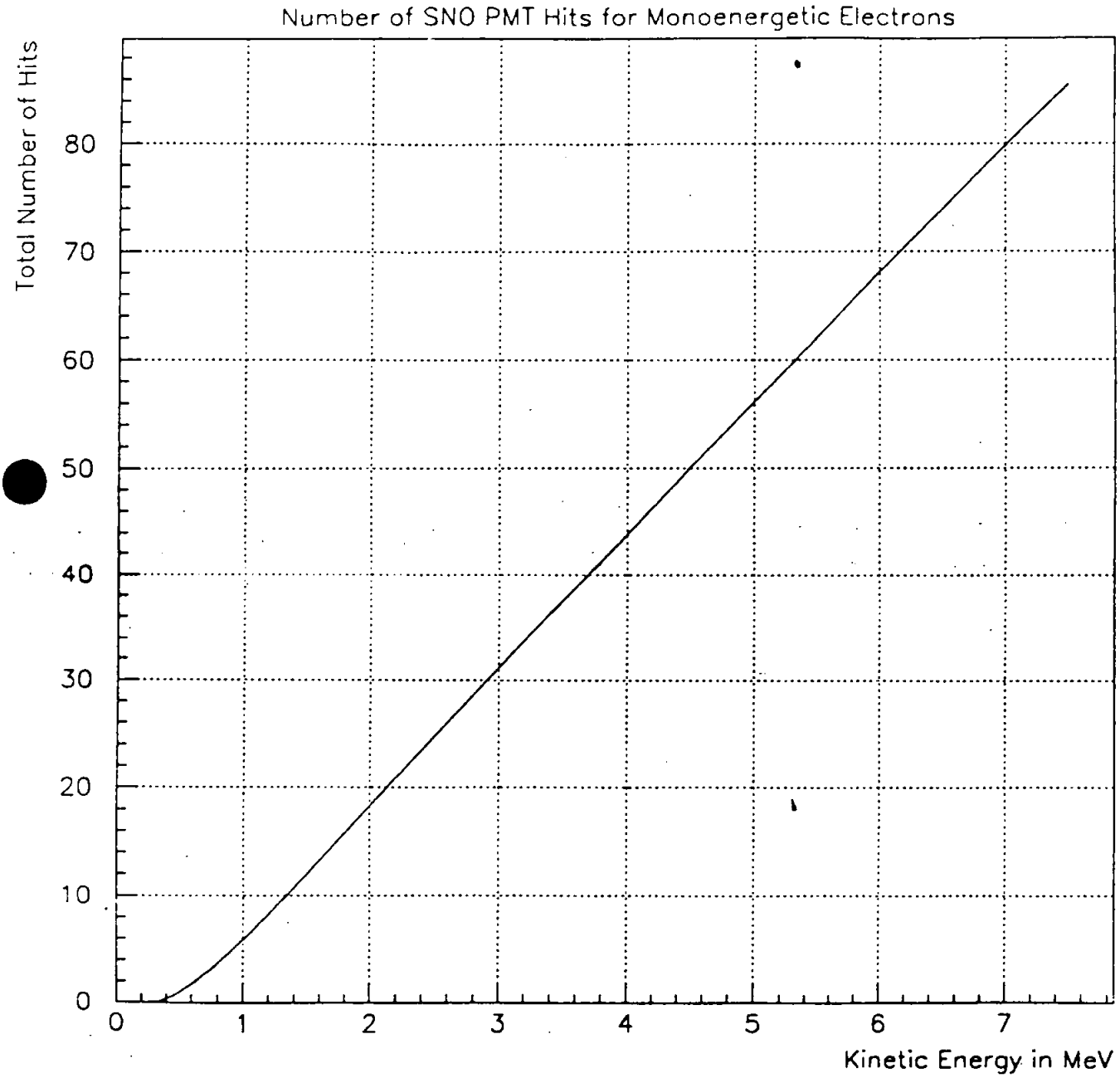
ρ=Density of compound in g/cm^3
Z=Effective atomic number of the compound
A=Effective atomic mass of the compound
k=Kinetic energy of electron in units of electron mass
I=Effective excitation potential of the compound
δ=Effective density effect parameter
Algorithm used is Equation 2.30 in LEO (W.R. Leo, "Techniques for Nuclear and Particle Physics Experiments", Springer-Verlag, 1987)
C=Shell correction, see LEO equation 2.33

-Energy loss is determined for compounds from elemental components using Bragg's Rule.
-Calculation of the plasma energy in the determination of the density effect is found in Sternheimer et al. Physical Review B. Dec 1982.
-Values of the Sternheimer parameters are found in the EGS4 manual (SLAC-265, Table 2.13.2) and in Sternheimer et al. Atomic Data and Nuclear Data Tables. Vol 30, March 1984.(Table 1).
-I is determined by using the semi-empirical formula equation 2.29 LEO.

b) Bremsstrahlung Energy Loss for No Screening:

$$-\left(\frac{dE}{dx}\right)_{Brem} = 4NEZ^2r_e^2\alpha\left[\ln\left(\frac{2E}{0.511}\right)-\frac{1}{3}-f(Z)\right]$$

N=Number density of compound in atoms/cm^3
r_e=Classical electron radius in cm
f(Z)=Correction to Born Approximation
(See LEO equation 2.67)


Number of SNO PMT Hits for Monoenergetic Electrons
Total Number of Hits
0
10
20
30
40
50
60
70
80
0
1
2
3
4
5
6
7
Kinetic Energy in MeV

b) Since this is an analytical calculation, not an event-by-event tracking of the photons, no conditions can be imposed on an event-by-event basis. For example in ANNEX6 a photon incident on the acrylic with an angle greater than 65 degrees will be reflected back into the acrylic such conditions can not be imposed in the calculation.

RESULTS:

The expected number of PMT hits is given in the figure for the mono-energetic electrons created at the centre. After 1 MeV the rise of PMT hits is approximately linear with a slope of 12 hits/MeV. In particular for a 7 MeV electron we get 80 PMT hits a simple estimate for the upper bound at that energy.