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Monte Carlo Study of Random PMT Mis-Alignment, SNO-STR-91-56 Preliminary

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1 Introduction

With the detailed engineering and costing efforts being done on the PMT support structure, the difficulty and consequent cost of achieving high levels of PMT-Reflector pointing accuracy have become clear. It is thus imperative to study the performance degredations that will be incurred for a given level of PMT mis-alignment in order to allow a rational selection of the alignment criterion.

This report attempts to do so by studying the change in the response of the current SNO design as a function of event radius when PMT's are randomly misaligned with probability distributions characteristic of alternative PMT mounting schemes. Section 2 details the input parameters to a modified Queens Monte Carlo used to carry out this investigation. The mis-alignment algorithm and probability distributions are discussed in section 3 and section 4 gives some simple reflector geometry considerations. The Monte Carlo results are presented in section 5. A brief discussion and summary of the conclusions which can be drawn are given in section 6.

2 Input Parameters

Monte Carlo runs of 10,000 7.5 MeV electrons were done using a version of the Queens code modified to include the Oxford reflector shape and Hamamatsu PMT shape as well as random mis-alignment. Electrons were given a uniformly random direction and an initial position uniformly distributed over a spherical shell set at various radii. The detector geometry was a 6 meter inner diameter spherical acrylic vessel, 5 cm thick, centered inside a PMT array of 9500 phototubes and reflectors. The center of the reflector opening was at at 8.5 meters before mis-alignment. Phototubes were arranged in concentric rings about the z-axis, equally spaced in theta, and the phi spacing adjusted to hold exactly the specified number of PMT's. The Queens code was modified so that the reflectors were quartic approximations to the 190 mm exit-diameter Oxford shape, perfectly aligned to the axis of the phototube, which was modeled with the actual Hamamatsu phototube shape of a central spherical portion and a circular toroidal edge. Absorption, scattering and quantum efficiency parameters are as in "Comparison of Input Parameters Used ... ", W. Frati, M. Lowry, P. Skensved, SNO-STR-91-24.

Events were fitted for position on the basis of arrival time. Fitting parameters are similar to those used in the past at Queens but with slightly less fixed cut of late arrival photons and more dependence on the chi-squared based cut. For the larger entrance aperture of the Queens reflectors, this results in a 5% increase in the number of fitted phototubes. The average number of fitted photons per event and the width of the distribution are the basis of the comparisons made here. Because the information on event location and direction is also derived from fitting the photon hit distribution, the variation in number of photons hits is intrinsicly connected to effects on position and angular resolution.

Event direction was determined from the average direction of all successfully fitted photons. This means the deduced direction is quite sensitive to asymmetric detection efficiency. It is probable that a geometry based method would be less sensitive but no such method has yet been implemented. Since we will probably produce a better direction determination algorithm in the future, no attempt has been made to study any angular effects with the current Monte Carlo.

3 Alignment Distributions

The Queens Monte Carlo code normally transports Cerenkov photons to the PMT sphere, determines the nearest PMT, and then transforms the photon coordinates to a local system with z-axis along the PMT axis. The photon is then tracked through the reflector-PMT geometry. If the photon scatters back out, the transformation back to the global coordinate system is made. For this study a version of the code was created with an additional coordinate transformation, specific to each phototube, being done and if necessary un-done.

This transformation corresponded to rotating the PMT-reflector assembly away from its alignment axis by a random amount specified at the beginning of the run and about a randomly selected point on the reflector rim, again specified at the start of the run. The rotation point was uniformly distributed about the rim but the amount of rotation followed one of the three distributions given in Table 1. A phototube was assigned to one of the angular ranges by the probability given in Table 1 and then uniformly assigned an angle within that angular range. These distributions roughly correspond to 1) the initial charge to the PSUP group, maximum mis-alignment of one degree, 2) cutting a geodesic panel into 4 pieces, placing the PMT's perpendicular to the quarter panel and then aligning the quarter panel, and 3) placing the PMT's perpendicular to a half panel and then aligning the half panel. Note that a half panel is the largest unit that will fit down the elevator shaft. For comparison purposes, runs were also done with perfect alignment of the PMT's.

4 Geometry

The reflectors serve a dual purpose, they gather more photons from the fiducial volume and fewer from outside it. They do this by imposing a limitation on the photon incident angle. A perfect 3-dimensional reflector for curved photocathodes is not possible but we can approximate the true performance with a perfect, sharp cutoff in angle at $\approx 58°$. Let us now look at the effect this has on photon detection from point-like events.

With out loss of generality we can assume the event occurs at a point z on the z-axis. Consider the angle made by a photon trajectory with the normal of the spherical distribution of phototubes at the point of interception, θ_{PMT}. This angle is essentially the incident angle of the photon on an intercepting reflector/phototube; the difference is always less than a degree for the reflector openings contemplated. If we further define the angle between the z-axis and the photon trajectory to be θ_{photon}, these two angles are related by:

$$z \sin\theta_{photon} = (8.5 \text{ meters}) \sin\theta_{PMT}.$$

For large enough z, θ_{PMT} will exceed the cutoff angle for θ_{photon} greater than some angle, θ_{vertex}, given by:

$$z \sin\theta_{vertex} = (8.5 \text{ m}) \sin 58° = 7.2 \text{ m},$$

where we assume the cutoff angle is 58°. For events inside 7.2 meters, all trajectories are detectable. For events outside, i.e. $z \geq 7.2$ meters, the family of detectable photon trajectories forms a solid, double cone with axis along z and with half-vertex angle θ_{vertex}. The undetectable trajectories form a band at 90° to the z-axis and centered on the event. For the largest radius considered here, 7.5 meters, $\theta_{vertex} = 74°$ and the band extends over 32°. We can also calculate the fraction of 4π in which photon trajectories are detectable, $f = 1 - \cos\theta_{vertex}$. At 7.5 meters, $f = 72.4\%$.

In our simplified picture, the effect of mis-alignment is to tilt some of the phototubes toward an event and some away. This is roughly equivalent to increasing or decreasing the cutoff angle. At 7.5 meters and for the maximum angle considered, $\pm 6°$, θ_{vertex} varies from 63° to 90° and the detection fraction from 55% to 100%. We can further approximate by saying that 20% of the phototubes have this mis-alignment with 1/4 tilted toward, 1/4 tilted away and the rest tilted to the side with no effect. This distribution would imply an increase in detection at $z = 7.5$ meters of $+0.5\% = 0.05 \times 100\% + 0.05 \times 55.0\% + 0.90 \times 72.4\% - 72.4\%$. Repeating the calculations for $z = 7.0$ m gives a drop of -1.5%.

We can contrast this geometry with that of a Cerenkov shower. The Cerenkov angle in water is 41.4° but multi-scattering of the electron smears this out. Plotted versus angle from the original direction, the distribution is roughly triangular between 0° and 90°, peaking at the Cerenkov angle. Thus the angular distribution of the photons is large in comparison to the detection cutoff band and we should not expect a marked increase in the width of the response due to event orientation.

The band of reduced efficiency will tend to skew the current fitter's direction determination toward the z-axis. Mis-alignment will either increase or decrease this skewing. However, for events in this region an even bigger effect is the skewing toward positive z caused by the presence of the acrylic vessel so that the degradation in directional information from mis-alignment would be minimal. Of course, we hope to eventually have a better direction finding algorithm and then we will need to return to this question.

One final point is that both the detection cutoff and Cerenkov shower geometries are much larger scale than the panel sizes in the geodesic dome structure. As a result, the random mis-alignments considered here should be an adequate approximation to the correlated mis-alignments introduced by aligning the phototubes to a portion of a panel.

5 Results

Table 2 presents the mean and variance of the fitted hits distribution for the eight event radii and the four alignment distributions. With 10,000 events the uncertainty in the mean and the variance should be about 1% of the variance. The 0.0 and 5.5 meter cases are intended to reveal the solid angle effect of turning the reflector opening. That is the major effect anticipated within the D_2O. The sequence of runs with source radii between 6.25 and 7.5 meters were done to reveal the changes due to the degradation in angular and thereby radial cutoff, which by the arguements in section 4 are expected to be concentrated here.

6 Discussion and Conclusions

The solid angle reduction is very small for the angles considered here and the results for inner events bears this out. There is no statistically significant change in mean or variance for events in the D_2O, i.e. at 0.0 and 5.5 meters.

In order for the radial cutoff to effect these events, the cutoff angle would have to be reduced by 13°, much greater than the amount considered here.

For the events outside the acrylic vessel the softening of the radial cutoff by the mis-alignment becomes evident. There is a drop in the average number of hits, reaching a maximum of 1.3% for the largest misalignment at a radius around 6.5 to 7.0 meters. There is no statistically significant change in variance at these radii.

As we proceed further out the sign of the change in response reverses, going through zero somewhere just beyond 7.25 meters. For events started at 7.5 meters (well beyond the nominal fiducial cut), there is an increase of $2.7 \pm .2\%$ in the number of hits and a weak indication of a small reduction in the variance. This fall and rise in average response is consistent with the simple picture presented in section 4.

On the basis of this data, there are several conclusions which can be reached.

1 There is no demonstrated effect on the CC and NC events at the 0.2% level in fitted hits.

2 The light water ES events would have a reduction in hits of order 1%, corresponding to a loss of about 100 PMT's. At $1500 per channel this would represent about $150k.

3 The events outside the fiducial volume will have a slightly enhanced response but the significance of this is difficult to quantify.

Overall the basic conclusion would have to be that this study has revealed no strong argument against a relaxation of the pointing specification to the maximum level considered here.

Table 1: Mis-Alignment Probability Distributions

label	0-1	1-2	2-3	3-4	4-5	5-6 degrees
1 degree	100.0%	*	*	*	*	*
4 degree	20.0%	60.0%	10.0%	10.0%	*	*
6 degree	1.1%	18.0%	53.9%	2.2%	21.3%	3.4%

Table 2: Fitted PMT hits [*mean* ± *variance*]

R(cm)	0 degree	1 degree	4 degree	6 degree
000	58.59± 9.01	58.66± 8.84	58.52± 9.06	58.55± 8.90
550	55.58± 8.78	55.33± 8.78	55.34± 8.82	55.32± 8.83
625	63.67±11.84	63.49±11.86	63.57±11.88	63.23±11.98
650	63.78±11.00	63.75±11.09	63.52±11.03	62.96±11.11
675	61.11±10.07	61.06±10.05	61.06±10.15	60.81±10.09
700	56.34± 9.24	56.51± 9.21	56.27± 9.19	55.69± 9.13
725	47.37± 8.00	47.25± 8.12	47.14± 8.09	47.12± 8.02
750	36.05± 7.77	36.03± 7.62	36.34± 7.55	37.01± 7.58