The Efficiency and Lifetime of Neutrons in the SNO Detector

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The efficiency and lifetime of a neutron in the SNO detector is determined by two factors, the probability of absorption of the neutron in the D_2O and the probability of escape from the D_2O. The lifetime is observable in cases where the time at which the neutron is created is known, such as a neutron created by a cosmic ray muon, or where a source produces several neutrons per decay and the time intervals between neutron-capture are determined. In the running phase of pure D_2O, the neutron is detected by its capture on the deuteron which yields a 6.25 MeV γ-ray. In this report we consider the two factors which yield the neutron efficiency and lifetime. We also present the neutron density distribution for a point source anywhere in the D_2O, and calculate the efficiency, mean lifetime and time distribution function for the point source as well as a uniformly distributed source. The effect of fiducial cuts is also analyzed. The neutron density distribution has already been presented in SNO-STR-98-002[1].

Absorption Lifetime: Consider an infinite vessel of D_2O plus a small contamination of H_2O. The neutron can be captured on deuterons, protons, ^{16}O and ^{17}O. Capture on ^{18}O is extremely small. The macroscopic cross-section for these processes is given by

$$\Sigma_A = N_A \sigma_A$$

where N_A is the number density of atomic species A and σ_A is its absorption cross-section. In the present estimate, we have used the neutron cross-sections from the Table of Isotopes, 8^{th} edition, which are given as thermal absorption cross-sections at a neutron speed of 2200 m/s. The values are

$\sigma_H = 0.333$ b, $\sigma_D = 0.519$ mb, $\sigma_{16_O} = 0.19$ mb and $\sigma_{17_O} = 0.236$ b. Hence, for SNO

$\Sigma \simeq (1 - 1.105\ f)\ \Sigma_{D_2O} + 1.105\ f\ \Sigma_{H_2O} + 7.84\ \delta_{17} \times 10^{-3}$ where f is the fractional contamination by mass of H_2O in the D_2O. The parameter δ_{17} is the difference between the fraction of ^{17}O in the D_2O and the assumed isotopic abundance of 3.8×10^{-4}. The macroscopic cross-sections are $\Sigma_{D_2O} = 4.38 \times 10^{-5}\ cm^{-1}$ and $\Sigma_{H_2O} = 2.23 \times 10^{-2}\ cm^{-1}$. In the same way, the macroscopic cross-section for the deuterons is $(1 - 1.105f) \times 3.45 \times 10^{-5}\ cm^{-1}$. The mean lifetime for a neutron is then given by

$$\tau = \frac{1}{\lambda} = \frac{1}{\Sigma \times 2.2 \times 10^5\ cm/s} \qquad \textbf{(1)}$$

Fig. 1 shows τ vs f for small values of f. For 99.92% D_2O, τ= 72 ms.

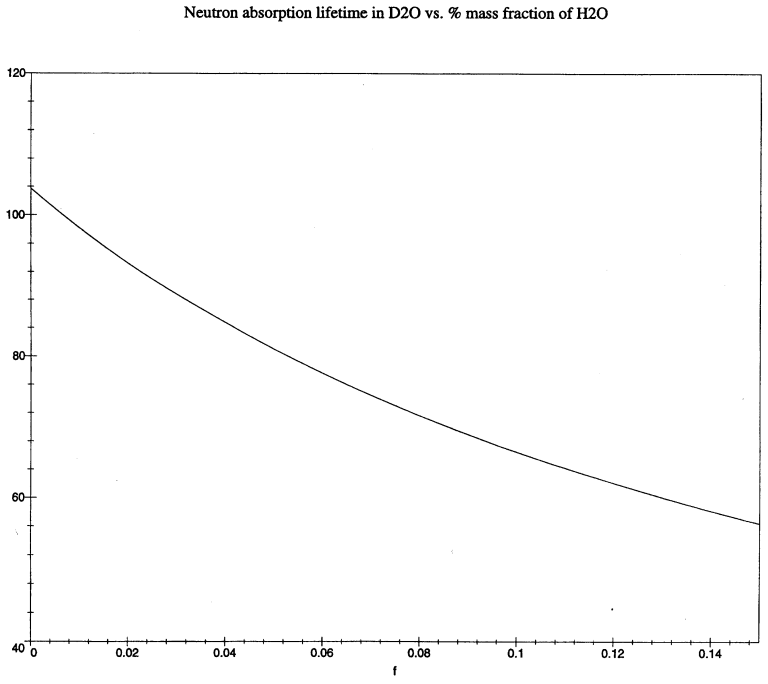

Fig. 1 Neutron absorption lifetime in ms in D_2O plus a mass fraction f of H_2O.

When NaCℓ is added to the D_2O, the lifetime will be considerably shortened because of the large cross-section for neutron capture on chlorine. If h is the amount of NaCℓ added by weight to the weight of heavy water, the macroscopic cross-section is

$$\Sigma = \frac{1}{1+h}\left[\Sigma_{water} + 0.509\ h\ \Sigma_{NaCℓ}\right] \qquad \textbf{(2)}$$

where Σ_{water} is the Σ for the appropriate enrichment of D_2O and $\Sigma_{NaCℓ} = 0.761\ cm^{-1}$. Fig. 2 shows the estimated lifetime for neutrons with salt concentration <0.25% in D_2O of 99.92% purity. For 0.1% salt concentration by mass, the neutron lifetime is about 10 ms.

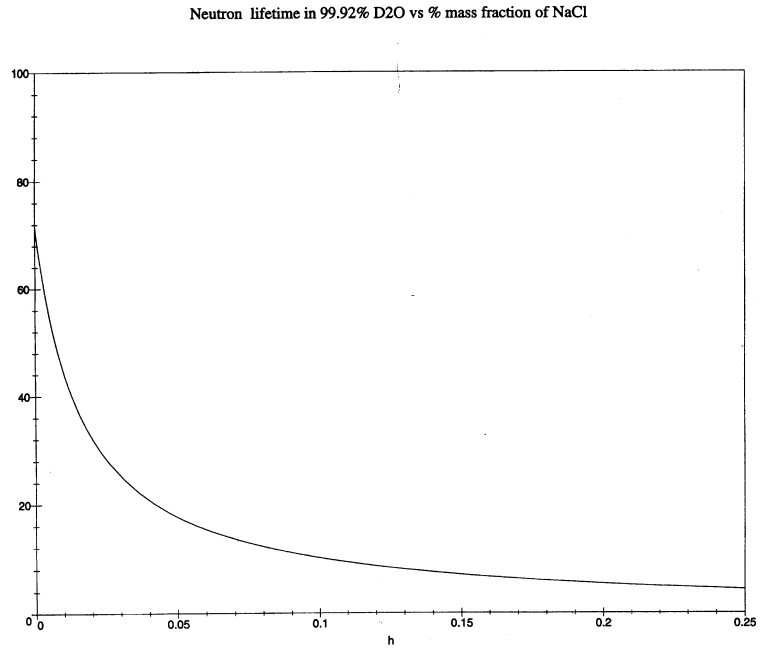

Fig. 2. Neutron absorption lifetime in ms in 99.92% D_2O plus a mass fraction h of NaCℓ.

Dissolved ^{14}N could also have an effect on the lifetime. Using a σ_{14_N} of 1.91b, the macroscopic cross-section for a ^{14}N concentration in D_2O of n g/g is $\Sigma_{14_N} = 0.0908n\ cm^{-1}$. If n were as large as 32.8 μg/g, then the ^{14}N would have an effect on the lifetime as large as 0.038% ^{17}O.

Diffusion length ℓ: The factor ℓ is called the diffusion length, and is given by

$$\ell = \sqrt{\frac{D}{\lambda}} = \sqrt{\frac{D}{\Sigma_a v}}\ . \qquad \textbf{(3)}$$

The diffusion constant D depends on the scattering cross-section, and because it is only about a factor of seven larger for H_2O than D_2O, it is not expected to change much with small H_2O concentrations. The value of D given by Etherington[2] for pure D_2O is $0.953 \times 2.2 \times 10^5\ cm^2/s$, thus giving a value of ℓ of 148 cm for pure D_2O and 123 cm for 99.92% D_2O. For H_2O we use $D_0 = 0.159 \times 2.2 \times 10^5\ cm^2/s$ and thus ℓ_0 is 2.67 cm.

In heavy water, the fermi age τ_F of thermal neutrons from a fission source is 125 cm^2 (ref. 2). In slowing down, the rms distance travelled by the neutron is $\sqrt{6\tau_F} = 27$ cm. However, this does not take very long, only about 4.6×10^{-5} s, for a neutron of initial energy of 2 MeV[3]. On the other hand, the slowing down makes a point source into a diffuse source. This has no impact on the lifetime of neutrons created near the centre of the D_2O; it may have some effect for neutrons produced near the acrylic vessel since neutrons can slow down into the acrylic and H_2O from the D_2O. To some extent a neutron produced in the H_2O could slow down into the D_2O, but the rms distance travelled by a fission neutron in H_2O is shorter, about 14 cm.

Neutron escape - general considerations: Since the thermal diffusion length ℓ for a neutron in 99.92% D_2O is very long, ~123cm, there is a high probability that a neutron, born somewhere at

random within the D_2O sphere, will in its thermal wandering reach the acrylic. There is a very high probability that it will then be captured on hydrogen in the acrylic (or in the light water) and be lost. This mechanism is an additional means of neutron absorption and will shorten the lifetime. We explore this quantitatively below by employing a diffusion approximation.

The time-dependent diffusion equation for the neutron distribution $\rho(\mathbf{r},t)$ can be written

$$\frac{\partial \rho}{\partial t} = -\lambda(\mathbf{r})\rho + D(\mathbf{r})\nabla^2\rho + \delta(t)\rho_0(\mathbf{r}) \qquad \textbf{(4)}$$

where both the decay rate λ and the diffusion constant D can vary with position. With the boundary condition that $\rho=0$ for $t<0$, we can identify $\rho_o(\mathbf{r})$ as the initial distribution of neutrons by integrating eq. (4) over an infinitesimal interval around t=0, from $t=0^-$ to 0^+. We also consider moments of $\rho(\mathbf{r},t)$,

$$\mu^{(k)}(\mathbf{r}) = \int_0^\infty dt\, t^k\, \rho(\mathbf{r},t)\ , \qquad \textbf{(5)}$$

and note that the zeroth moment $\mu^{(0)}(\mathbf{r}) = \rho_s(\mathbf{r})$ is a solution of the time-independent diffusion equation with source $\rho_o(\mathbf{r})$. That is,

$$0 = -\lambda(\mathbf{r})\rho_s + D(\mathbf{r})\nabla^2\rho_s + \rho_0(\mathbf{r}) \qquad \textbf{(6)}$$

as can be shown by integrating eq. (4) over all time from $t=0^-$ to ∞.

In the case of spherical symmetry, the time-dependent diffusion eq. (4) can be solved by expanding the neutron density in terms of spatial eigenmodes

$$\rho_n = \frac{A_n}{r} \sin\left(\frac{n\pi r}{R_e}\right) , \; n = 1,2\ldots , \tag{7}$$

here written to accommodate the acrylic (and/or light water) by a linear extrapolation model[3]. The extrapolated radius R_e is the point where a perfect absorber representing the effect of the acrylic (and/or light water) is to be placed. It is

$$R_e = R + D\ell_0/D_0 \tag{8}$$

which is 16cm beyond R for H_2O and about 14.5cm beyond R for acrylic. The time dependent amplitudes A_n satisfy

$$\frac{\partial A_n}{\partial t} = -\lambda\left(1 + \frac{n^2\pi^2\ell^2}{R_e^2}\right) A_n \tag{9}$$

with solution

$$A_n = \alpha_n \, e^{-\lambda t\left(a^2 + n^2\pi^2\ell^2/R_e^2\right)} \tag{10}$$

where in eq. (10) we have introduced a parameter *a* which will ultimately be set to unity but is convenient for generating the moments $\mu^{(k)}$ by differentiation. The constants α_n must be chosen so that at time t=0 the distribution $\rho(\mathbf{r},t=0)$ is the source term $\rho_o(\mathbf{r})$. For the case of a spherical shell source at radius s,

$$\alpha_n = \frac{1}{2\pi s R_e} \sin\left(\frac{n\pi s}{R_e}\right) \tag{11}$$

so that the complete solution for the neutron distribution is

$$\rho(\mathbf{r},t) = \frac{1}{2\pi r s R_e} \sum_{n=1}^{\infty} \sin\left(\frac{n\pi s}{R_e}\right) \sin\left(\frac{n\pi r}{R_e}\right) e^{-\lambda t\left(a^2 + n^2\pi^2\ell^2/R_e^2\right)} \tag{12}$$

The normalization in eqs. (11,12) is such that $4\pi\int r^2 dr \rho_o = 1$ independent of s. That is, at time t=0,

$$\rho(\mathbf{r}, t = 0) = \rho_0(\mathbf{r}) = \frac{\delta(r-s)}{4\pi r s} \; . \tag{13}$$

The time integration defining the zeroth moment leads to the steady state solution

$$\rho_s = \mu^{(0)} = \frac{1}{2\pi\lambda r s R_e} \sum_{n=1}^{\infty} \frac{\sin\left(\frac{n\pi s}{R_e}\right) \sin\left(\frac{n\pi r}{R_e}\right)}{a^2 + \left(\frac{n^2\pi^2\ell^2}{R_e^2}\right)} \tag{14}$$

and the summation can be carried out explicitly by first transforming to an equivalent reciprocal-space sum. The result is

$$\mu^{(0)} = \frac{1}{4\pi\lambda r s a \ell \sinh\frac{aR_e}{\ell}} \begin{cases} \sinh\frac{a(R_e - s)}{\ell} \sinh\frac{ar}{\ell} & r < s \\ \sinh\frac{a(R_e - r)}{\ell} \sinh\frac{as}{\ell} & r > s \end{cases} \tag{15}$$

and the higher order moments are trivially generated by differentiation as

$$\mu^{(k)} = \left[-\frac{1}{2a\lambda}\frac{\partial}{\partial a}\right]^k \mu^{(0)} \Big|_{a=1} \; . \tag{16}$$

The formulae (12-16) form the basis for all the subsequent analysis. However, in general the source

will be distributed and so the expressions for $\rho(\mathbf{r},t)$ and $\mu^{(k)}(\mathbf{r})$ are to be replaced by averages over the source positions. For example, for a distribution uniform over the tank, we need to make the replacements

$$\rho(\mathbf{r},t) \to \frac{3}{R^3}\int_0^R s^2\, ds\, \rho(\mathbf{r},t) \quad , \quad \mu^{(k)}(\mathbf{r}) \to \frac{3}{R^3}\int_0^R s^2\, ds\, \mu^{(k)}(r) \; . \tag{17}$$

One further integration will yield the total number of neutrons in some volume. We denote this neutron count and its moments in the volume, not necessarily the full volume of the D_2O sphere, by

$$N(t) = \int d^3r\, \rho(r\,,\,t) \quad , \quad M^{(k)} = \int d^3r\, \mu^{(k)}(\mathbf{r}) = \int_0^\infty dt\, t^k\, N(t) \; . \tag{18}$$

Since neutrons are lost by various mechanisms and both the mechanisms and neutron distributions are inhomogeneous in space, the "lifetime" of the neutron depends on the details of the measurement process. For the experimental situation in which the lifetime is determined by recording capture on deuterium, one can argue that the event rate is directly proportional to the neutron density in time and thus that the mean lifetime, here called τ_c for capture, is just the ratio

$$\tau_c = M^{(1)}/M^{(0)} = -\frac{1}{2\lambda}\frac{\partial}{\partial a}\ell n\, M^{(0)}\,|_{a=1} \; . \tag{19}$$

An efficiency can be defined as the fraction of neutrons capturing on D,

$$\epsilon = \lambda_D \int_0^\infty \frac{N dt}{N(t=0)} = \lambda_D \frac{M^{(0)}}{N(t=0)} \tag{20}$$

where $\lambda_D \approx 7.58/s$ is determined by eq. 1.

The remaining sections describe various specializations of the diffusion model formulae in this section.

Neutron distribution in time: Since the extrapolation length $D\ell_o/D_o$ in eq. (8) is small, the qualitative features of the neutron distribution in time can be understood just as well by setting R_e=R. This is not an essential approximation; rather it is used here just to keep the quoted formulae manageable.

For the special case of a source in the middle of the D_2O sphere we need only take the limit s→0 in the formulae of the preceding section. If the integration volume in eq. (18) is taken as the entire sphere we find, starting with eqs. (12,13),

$$N(t) = 2 \sum_{n=1}^{\infty} (-1)^{n-1} \exp^{-\lambda t(1+n^2\pi^2\ell^2/R^2)} \quad , \quad N(t=0) = 1 \ . \qquad \textbf{(21)}$$

It is seen that the neutron density does not decay with a single exponential, so that the mean lifetime (cf. eq. (26) below) deduced from eq. (19) is the appropriate lifetime to quote. Thus one should not fit an exponential to the data to determine the neutron lifetime but rather determine the mean life from the data. In order to prevent biasing the result, one should use a time duration of about 6 times the mean life in order that the observed mean life not be less than about 98% of the true mean life. (This estimate is only approximate, being based on the assumption of a single exponential lifetime $\exp(-\lambda t)$.)

In the case of a uniform source throughout the vessel, again with perfect absorber at radius R,

$$N(t) = \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp^{-\lambda t(1+n^2\pi^2\ell^2/R^2)} \quad , \quad N(t=0) = 1 \ . \qquad \textbf{(22)}$$

This case is somewhat pathological because -dN/dt becomes infinite as t→0. Note that this arises from that population of neutrons that originate near the vessel boundary and diffuse towards it in a short time. The use of the more realistic extrapolation approximation with R_e>R would correct this deficiency.

Point source at radius s: Provided that neutrons are detected without regard to direction from the sphere center, the point source can be replaced by a spherical shell at radius s and the formulae of the earlier theory section are applicable. Again to keep simple the quoted expressions below we specialize to an absorber at radius R=6m but give selected results also for the extrapolation approximation with R_e=6.16m.

If all neutrons at radius r<R are detected, the integration of eq. (15) leads to

$$\lambda M^{(0)} = \frac{1}{a^2}\left[1 - \frac{\frac{R}{s}\sinh\frac{as}{\ell}}{\sinh\frac{aR}{\ell}}\right] \qquad \textbf{(23)}$$

Since N(t = 0) = 1, $M^{(0)}$ with the parameter a = 1 becomes the expression for the efficiency given in eq. (20), i.e.

$$\in = \frac{\lambda_D}{\lambda}\left[1 - \frac{\frac{R}{s}\sinh\frac{s}{\ell}}{\sinh\frac{R}{\ell}}\right]. \qquad \textbf{(24)}$$

Using the improvement of an extrapolated distance R_e, the efficiency is

$$\epsilon = \frac{\lambda_D}{\lambda}\left\{1 - \frac{R}{s}\,\frac{\sinh\left(\frac{s}{\ell}\right)}{\sinh\left(\frac{R_e}{\ell}\right)}\left[\cosh\left(\frac{R_e - R}{\ell}\right) + \frac{\ell}{R}\sinh\left(\frac{R_e - R}{\ell}\right)\right]\right\} \qquad \textbf{(25)}$$

Fig. 3 shows the efficiency calculated using eq. (25) for a central point source (s = 0) for different values of D as a function of absorption cross-section Σ_a. (A cut excluding neutrons at distances less than 1 m from the source is included - see following section.)

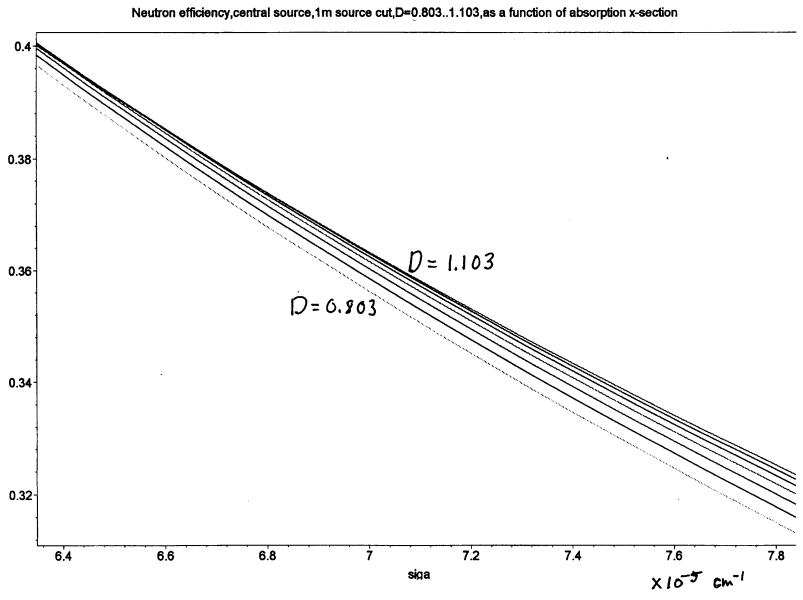

Fig. 3. Efficiency of neutrons from a central point source, with a cut excluding events less than 1 m from the source, as a function of the absorption cross-section Σ_a in units of 10^{-5} cm^{-1} for different values of the diffusion constant D from 0.803 cm (bottom curve) to 1.103 cm (top curve) in steps of 0.05 cm.

Eq. (23) also directly gives the mean lifetime reduction factor

$$\lambda\tau_c = 1 + \frac{1}{2}\,\frac{\frac{R}{\ell}\cosh\frac{R}{\ell}}{\sinh\frac{R}{\ell}} - \frac{1}{2}\,\frac{R}{\ell}\,\frac{\left[\cosh\frac{R}{\ell} - \cosh\frac{s}{\ell}\right]}{\left[\sinh\frac{R}{\ell} - \frac{R}{s}\sinh\frac{s}{\ell}\right]} \qquad \textbf{(26)}$$

by performing the differentiation indicated in eq. (19). The lifetime from eq. (26) for a source in the centre for various diffusion lengths is shown as a function of Σ_a in fig. 4.

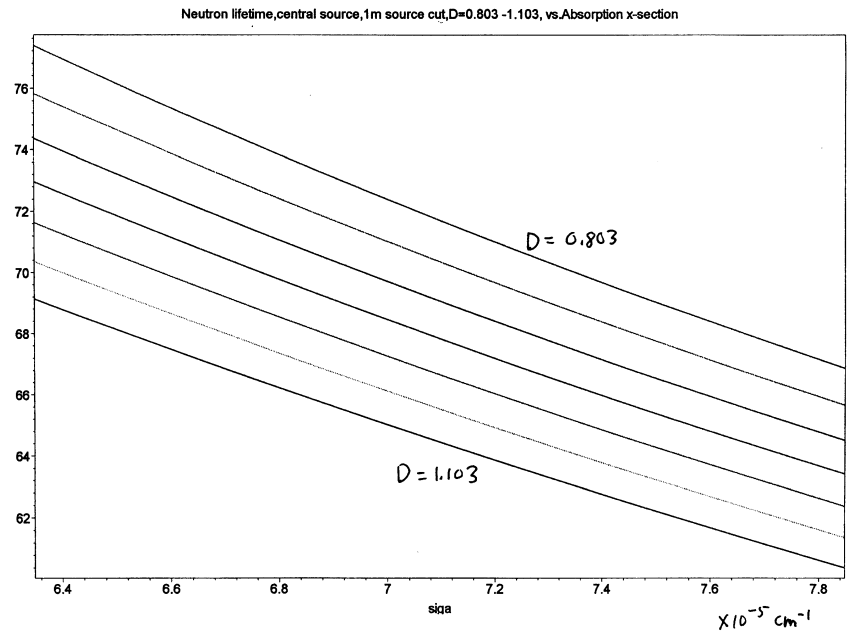

Fig. 4. Lifetime of a neutron (in ms) from a central point source, with a cut excluding events less than 1 m from the source as a function of the absorption cross-section Σ_a in units of 10^{-5} cm^{-1} for different values of the diffusion constant from 0.803 cm (top curve) to 1.103 cm (bottom curve) in steps of 0.05 cm.

Calculations have also been performed in which the exact eigenmodes for D_2O surrounded by light water are used in place of the approximate modes in eq. (7). Our conclusion is that the extrapolation approximation is very good and entirely adequate. Some comparative numbers for the lifetime reduction factor are 0.8444, 0.8556, 0.8558 for the source at the centre, 0.5388, 0.5671, 0.5675 for s = 5 m and 0.3718, 0.4342, 0.4320 for s = 6 m; the factors are respectively eq. (26), the prediction of the extrapolation approximation, and the diffusion calculation with exact eigenmodes. Corresponding numbers for the efficiency with $\lambda_D/\lambda = 0.543$, appropriate for an assumed isotopic abundance of ^{17}O of 3.8×10^{-4}, are 0.5027, 0.5064, 0.5065 for the source at the centre, 0.2541, 0.2805, 0.2811 for s = 5 m and 0, 0.0496, 0.0509 for s = 6 m; these values are eq. (24), eq. (25), and the diffusion calculation with exact eigenmodes respectively. The most dramatic differences are for the source $s \approx 6$ m; note that $\epsilon \approx 0$ shows few neutrons escape absorption in this instance while $\tau_c \approx 0.37/\lambda$ from eq. (26) implies that the small fraction that escape immediate absorption and diffuse to smaller radii have a finite mean lifetime.

Fiducial Cuts - Point Source at Centre: A fiducial cut on the 6.25 MeV γ-events will affect the observed lifetime and efficiency. It is relatively straightforward to model a fiducial cut for a point source at the centre. The relevant moment which excludes events both within a radius r_1 and beyond a radius r_2 is

$$M^{(0)} = \int_{r_1}^{r_2} r\, dr\, \frac{\sinh\left(a\dfrac{R_e - r}{\ell}\right)}{\left[\lambda \ell^2 \sinh\dfrac{aR_e}{\ell}\right]} . \qquad \textbf{(27)}$$

Lifetime and efficiency follow from the prescriptions in eqs. (19) and (20). As an example, for a cut excluding captures at r < 1 m, the efficiency deduced from eq. (27) is 0.4000 compared to 0.5064

from eq. (25) for s = 0. We cannot easily model such source cuts if the source is not in the centre ($s \neq 0$) because of the breakdown of spherical symmetry but a reasonable approximation would be to assume the same reduction factor $0.4000/0.5064 \approx 0.79$ for all s. A comparison of efficiency with this approximation to that obtained by MC by Mark Boulay is shown in fig. 5.

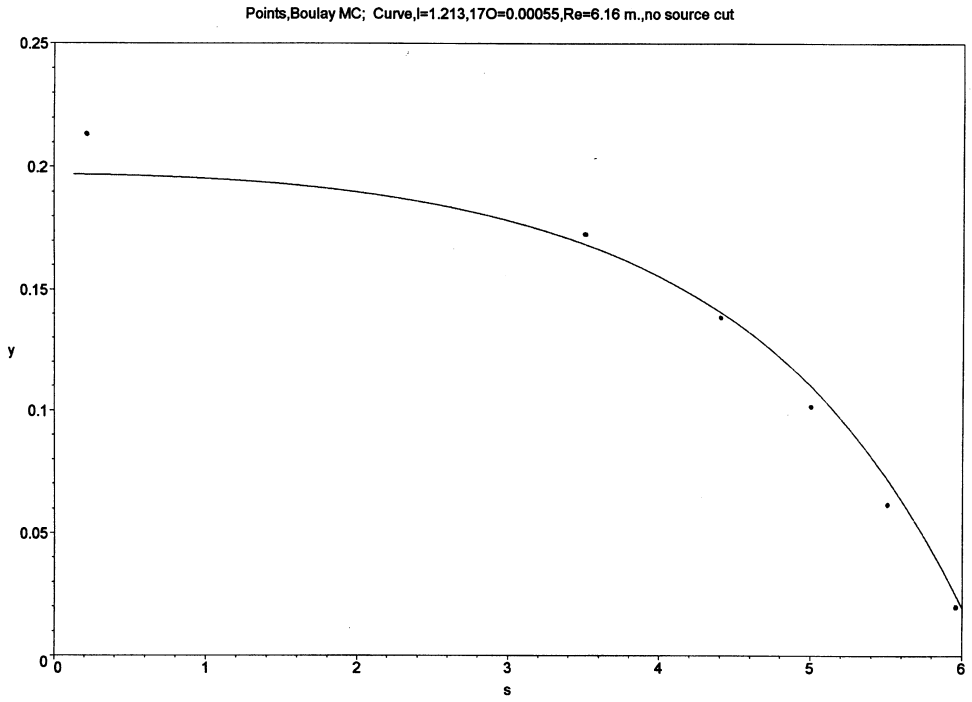

Fig. 5. Efficiency as a function of distance from a central source. The source was 0.216 m from the centre. The curve is the analytic expression for the efficiency (eq. (25)) with no fiducial cut around the source but rescaled by a constant factor as described in the text. An extrapolated distance of 6.16 m was used and a λ_D/λ of 0.532. The points are Mark Boulay's MC code, with a 1 m source cut. Both calculations have been normalized by a further factor of ½ to account for an Nhit cut at the peak of the 6.25 MeV γ-ray.

Note that a further factor of ½ has been included to account for the Nhit cut at the peak of the 6.25 MeV γ-ray. The discrepancy at s = 0 is at least in part due to the fact that our calculation only models the neutron diffusion and does not include the distance travelled by the 6.25 MeV γ-ray before absorption. Neither does it include the radial uncertainty in the event reconstruction. If both of these effects are included as a gaussian smearing of rms length σ, then one can show that the efficiency we calculate should be increased by the factor $\exp(\sigma^2/(6\ell^2)) \approx 1.03$ for σ = 0.5 m. (This holds provided the source cut is larger than about 2σ which is true here.)

Fiducial Cuts - Uniform Source: We consider the case in which only those events inside some radius $R_f \leq R$ are included in the lifetime measurement. The zeroth moment is

$$M^{(0)} = \frac{1}{\lambda a^2 R^3}\left\{R_f^3 - 3\left(\frac{\ell}{a}\right)\left[R_f \cos\left(\frac{aR_f}{\ell}\right) - \left(\frac{\ell}{a}\right)\sinh\left(\frac{aR_f}{\ell}\right)\right] - \frac{\left[R\cosh\left(a\frac{R_e-R}{\ell}\right) + \left(\frac{\ell}{a}\right)\sinh\left(a\frac{R_e-R}{\ell}\right)\right]}{\sinh\left(\frac{aR_e}{\ell}\right)}\right\} \quad \textbf{(28)}$$

and we find that the efficiency $\lambda_D M^{(0)}|_{a=1} = 0.302$ with $R_f = R$. Also, the lifetime reduction factor $\lambda\tau_c$ deduced directly from eq. (28) by differentiation is 0.6572 for $R_f = R$.

References:

1. J.J. Simpson and J.-X. Wang, Determination of NC/CC Ratio and CC shape during the pure D_2O State (Part II), SNO-STR-98-002.
2. H. Etherington, Nuclear Engineering Handbook. First Edition, McGraw-Hill, 1958.
3. S. Glasstone and M.C. Edlund, The Elements of Nuclear Reactor Theory, D. Van Nostrand, 1958.