An Improved Grid Fitter

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Abstract

Some major modifications have been added to the grid fitter. This report presents evaluations of the new grid fitter. We show that this fitter is better than the time fitter by every criteria except the speed. More importantly, the fitter may be more accurate on reconstructing PMT beta gammas than any exsiting fitters in SNOMAN. The report also presents some subtle problems encountered during development of the fitter.

1 Motivation

Arguably vertex fitter is one of the most important tools in SNO data analyses as it enables us to figure out where events occured. By doing so, we are able to reject most of external background events whose number is normally much higher than the number of detected neutrino events. Better reconstruction accuracy also enhances our capability to analyze hit patterns of various classes of events which is the foundation of the pattern recogntion techniques developed to carry out CC/NC and Bi/Tl separation. There are several vertex fitters already existed in SNOMAN, of which the time fitter is the most frequently used one because it is fastest. The grid fitter has been shown [1] by Monte Carlo data to be the best fitter to minimize reconstruction errors. However, as the grid fitter was optimized according to MC data which always assume a perfectly working detector, it may not be applicable to the real data. Some improvements have been implemented by the author to enhance performance of the grid fitter.

Fitting event direction is also very important in data analyses, especially for CC or ES events as their directions have corelations with source of the detected neutrinos, the sun. The direction information should give us another efficient way to reject radioactive backgrounds which are always isotropic. A well known shortcoming of the existing fitters in direction reconstruction is that they don't use the fact that many Cerenkov photons produced by an electron are emitted along a Cerenkov cone. We tried to take advantage of this characteristic information in the reconstruction by using a maximum likelihood method.

During this paper, we will try to argue that this improved grid fitter is better than the time fitter in both position and direction reconstruction and should be used in the data analyses.

2 Vertex Reconstruction

Among all the existing fitters, the time fitter is the simplest and the most frequently used one. It works by minimizing

$$\chi^2 = \sum_i \frac{(t_i - t_0 - \frac{d_i}{c'})^2}{\sigma^2} \tag{1}$$

$$d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \tag{2}$$

, where x_i, y_i, z_i and t_i are the spatial and temporal coordinates of the i^{th} hit in an event; x_0, y_0, z_0 and t_0 are the spatial and temporal coordinates of the source which are the paramters to fit while σ is the width of the PMT transit time . During the minimization, hits not in time are thrown out one by one until all the remaining hits fall inside a pre-defined time window. In the language of maximum likelihood fitting, the time fitter essentially uses a Gaussian function as the probability distribution function.

The grid fitter enhances its performance by applying a p.d.f. optimized according to results of Monte Carlo simulations. Because of this, it is not surprising to see that when applied to MC data, the grid fitter works better than the time fitter. However, the most powerful aspect of the grid fitter comes from its grid search during the minimization. The time fitter always uses the center of the detector as the starting point for minimization, which is susceptible to the local minimum. By sampling grid points in the parameter space, the grid fitter is much more robust to find the global minimum.

As the p.d.f.s for real data are unknown, it may be imprudent to directly use the p.d.f.s generated by MC in the maximum likelihood fitting for the real data. Besides, it is not appropriate to use the p.d.f. of a particular class of event which may lead to bias in the reconstruction. As shown in equation 1, a out-of-time hit has a much larger χ^2 value than an in-time hit, thus a few out-of-time hits caused by scattering or noise could have large effects on reconstructed positions. In order to minimize affects of these so called bad hits, a truncated Gaussian is better than a full Gaussian as the p.d.f. because for the truncated Gaussian, any hits outside a predefined window can only have a constant contribution to the likelihood. Other factor need to be considered is that there are normally more late hits due to reflections/refractions than earlier hits due to noises. It is difficult, if not impossible, to evaluate ratio between no. of late hits and no. of early hits. We chose an asymetrically truncated gaussian as the p.d.f. to take into account such affect. In practice, we chose (-6σ, +3σ) as the predefined time window while $sigma$ is roughly equal to width of the PMT transit time.

There are also suggestions that angular information of the detected photons could be used in the vertex reconstruction. If photons are produced by an electron, many of photons are emitted on a Cerenkov cone along the electron direction. This extra information could, in principle, improve reconstruction accuracy. We tested this theory by including angular information in the likelihood function, the results didn't show tangible improvement. Besides, we are not able to know whether an event is a single electron or a multiple particle event before the reconstruction. For example, reconstructing a neutral current event with assumption of a single electron will almost certainly lead to a worse result. Thus we decided not to couple the temporal and the angular information together.

Several additions to the fitter are worth mentioning:

- One technique applied to remove hits which are obviously out of time is to compare difference in time between a pair of hits with distance between the two hits. If a pair of hits are generated by straight line travelling photons, their relative time $\triangle_{ij}$ and distance d_{ij} should satisfy: $\triangle_{ij} < d_{ij}/c$, otherwise either one or both hits are bad. We evaluate all pissble pairs of hits in an event against such equation and record times of failures for each hits. In the end, any hits with number of failures larger than 0.5*nhits is purged from later reconstruction. In pratice, in order to account for the PMT jitter time, we use equation $\triangle_{ij} < d_{ij}/c + t_{tol}$. t_{tol} is set at 10ns, but it may be set at a smaller value.

- Another trick added to the grid fitter is to impose a sliding window on the hits before the reconstruction. Width of the window is chosen to be 76ns, which is the time it takes for a photon to traverse the whole detector. The window is slided along the time axis to select a cluster of hits so that the number of hits falling inside the window is maximized. Any hits outside this window is purged from later reconstruction. This trick may help to remove the bad hits. This sliding window is optional because of some problems we encountered when it was applied on the real data, as shown later.

- We use the position found by the grid search as starting position to minimize the chi square value based on a truncated gaussian p.d.f., and then throw out the hit with largest chi square value. Such process is iterated until all of the remaining hits lie within a predefined assymetric window. This process, which is similar as what is used in the time fitter, may help to remove some bad hits through iterations.

3 Direction Reconstruction

Definition of the direction is only meaningful for a single particle event. Thus the idea of applying Cerenkov cone information is valid for direction reconstruction. Again the realistic p.d.f. is unknown without data from an electron calibration source. Several thousand 5Mev electrons were generated by SNOMAN to produce such p.d.f. with results shown in figure 1. Two polynomial function were used to fit the distribution:

$$prob(\theta) = \begin{cases} -7.4809 + 1354.3 * \theta - 1617.6 * \theta^2 + 3611.6 * \theta^3 & \theta < 0.733 \\ 4742.3 - 6657.1 * \theta + 3674.4 * \theta^2 - 925.23 * \theta^3 + 88.222 * \theta^4 & \theta > 0.733 \end{cases} \tag{3}$$

where θ is the angle between the photon direction and the electron direction. Thus the event direction is reconstructed by maximizing the following function:

$$Likelihood = \sum_i log(prob(\theta_i)) \tag{4}$$

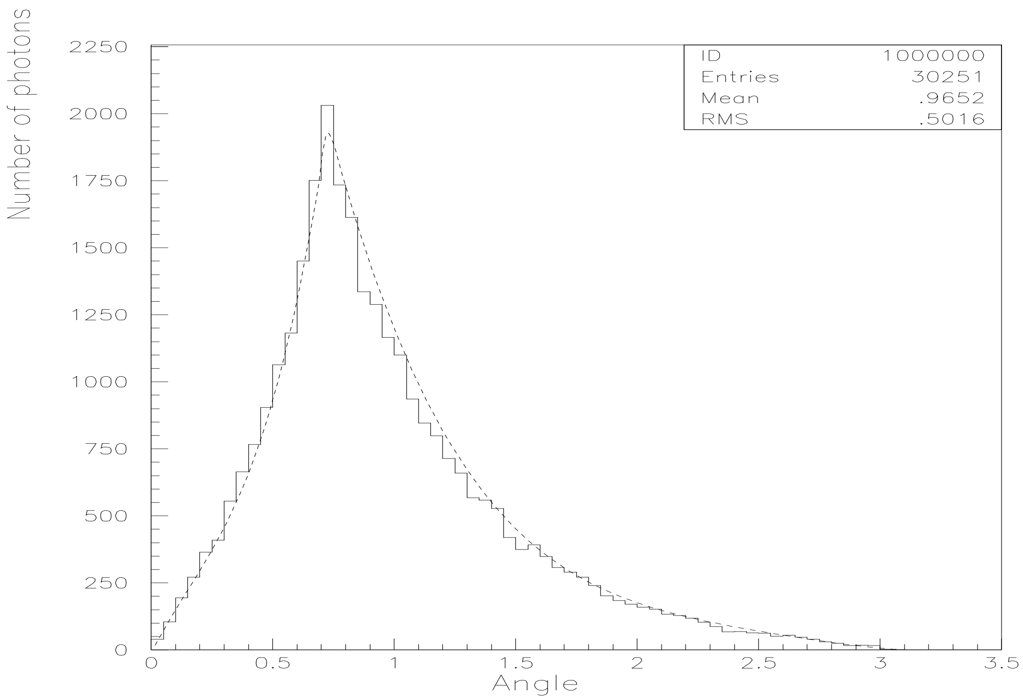

Figure 1: Angular distribution of Cerenkov photons produced by 5Mev electrons uniformly generated in the D_2O region. Dashed curve is the probability function used to fit the distribution

. In practice, the hits thrown out by the vertex reconstruction are not used in the direction reconstruction.

We also carried out studies to evaluate how sensitive the fitted results are against shape of the p.d.f. used in the maximum likelihood fitting. By replacing the p.d.f. produced by 5Mev electrons with the p.d.f. produced by 10Mev electrons and even with a truncated Gaussian peaked at Cerenkov angle, there were no tangible change on the fitted direction. On the other hand, we may need an electron source to draw a final conclusion.

The fitter also has the option to fit the event direction by simply averaging directions of photons. This may help studying general topologies of events when they don't necessarily consist of a single particle.

4 Reconstruction Results

4.1 Monte Carlo Data

Monte Carlo is always the first step to quantitize performance of the fitter as both position and direction of a simulated event are known and can be compared with reconstruction results.

Both the time fitter and the grid fitter have been applied to a set of 5000 5Mev electrons generated uniformally in the heavy water by SNOMAN. As shown in figure 2, the improved grid fitter reconstructed roughly 5% more events within 2 meters

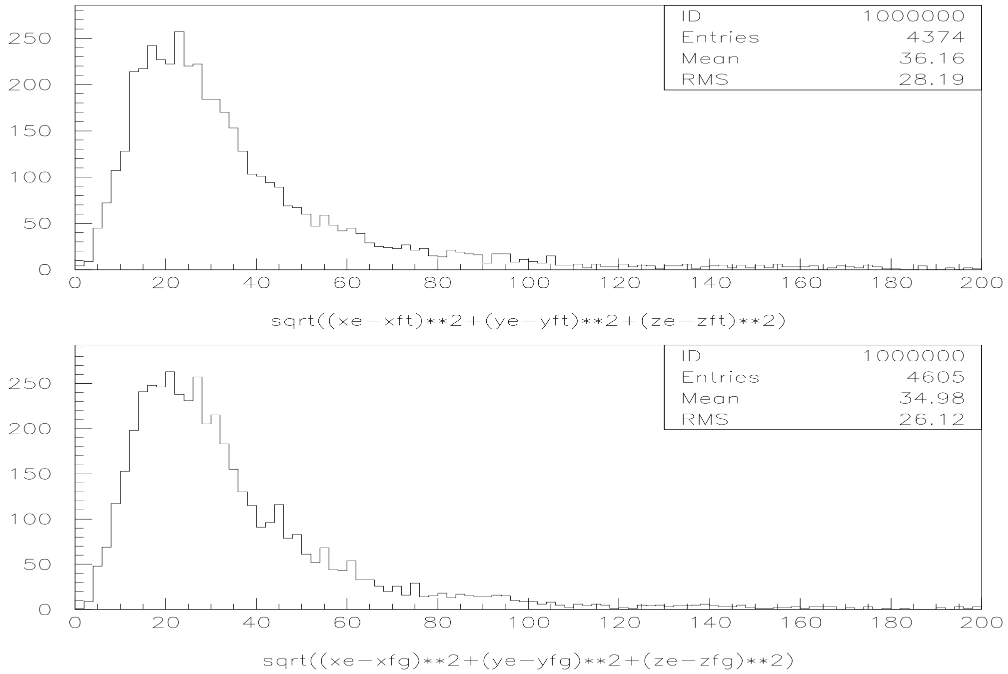

Figure 2: Rdev (distance between the fitter vertex and the actual vertex) distributions for 5000 5Mev electrons uniformally generated in the heavy water by SNOMAN. Upper plot is result of the time fitter while the bottom plot is result of the improved grid fitter. A nhits cut of $nhits \geq 30$ is imposed.

of their source positions than the time fitter. Both width and mean of the Rdev distribution generated by the improved grid fitter is smaller than the plot generated by the time fitter. Figure 3 shows errors on the fitted direction for the time fitter and the grid fitter. Clearly the improved grid fitter works much better than the time fitter.

Vertex reconstruction is also very important for low Nhits events. At low nhits, radioactive background events are dominant, thus vertex reconstruction is much more important than direction reconstruction. Among all radioactive decays, the ^{208}Tl and the ^{214}Bi decay are the most important ones. We applied the fitters to ^{208}Tl decay events generated by the SNOMAN code with results shown in figure 4. Again the improved grid fitter works better than the time fitter. Another less important criterion to evaluate performance of a fitter is its efficiency. Table 1 shows success rate of two fitters on various classes of MC events. The improved grid fitter is also better than the time fitter in this category.

The most difficult problem in background study for SNO is how to reject and estimate number of events caused by radioactive decays in ≈ 10000 PMTs. Main cause of its difficulty is just that the number of such type of events is huge. The vertex fitter is one of the most important tools to reject PMT beta gamma events. Data from the detector seems to show there are more than expected number of events occuring in the PSUP region. Thus, in our opinion, number of PMT beta gamma events mis-reconstructed into the heavy water region is a very important criterion

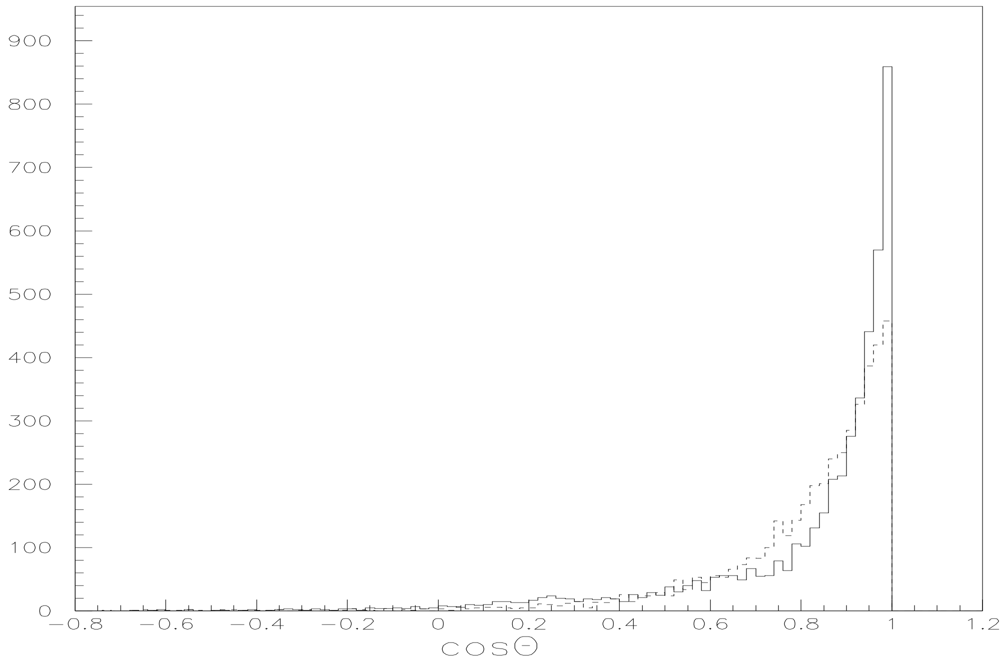

Figure 3: cosθ distributions generated by the time fitter (dashed line) and the improved grid fitter (solid line) for 5000 5Mev electrons uniformally distributed in the heavy water. θ is the angle between the actual electron direction and the fitted direction. A cut of $rdev < 200cm$ and $nhits \geq 30$ is imposed. Maximum likelihood method was used in the direction fitting.

Table 1: Efficiency of fitters on MC events, with a cut of $nhits \geq 30$.

		The time fitter	the improved grid fitter
5 Mev electron	OK	4667	4717
	Failed	51	1
^{208}Tl decays	OK	6889	7023
	Failed	211	77

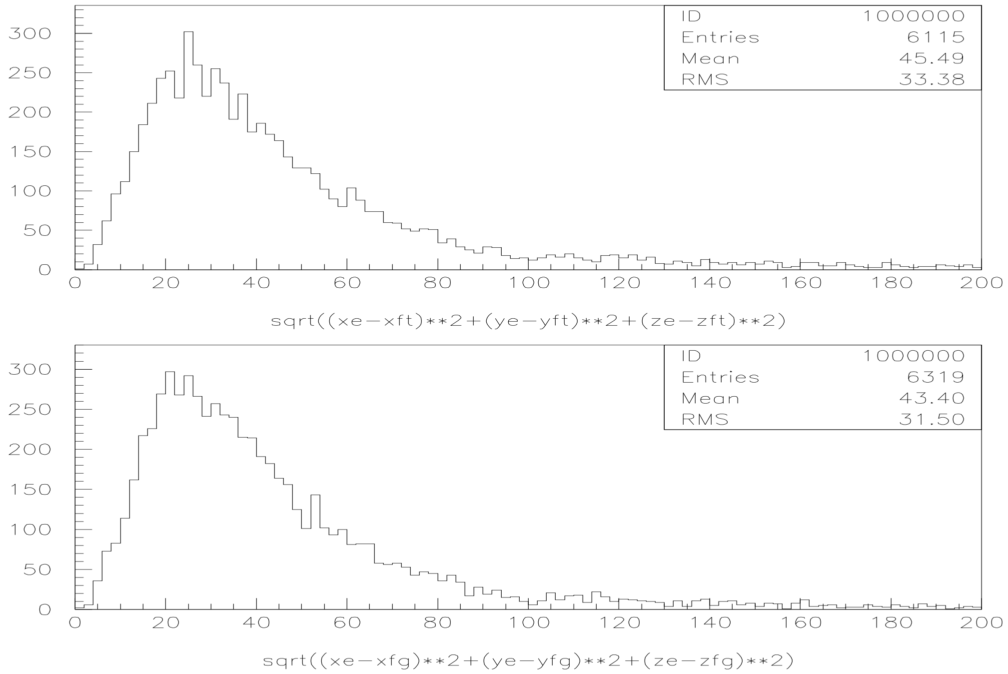

Figure 4: Rdev distributions of ^{208}Tl decay events uniformally distributed in the D_2O fitted by the time fitter (upper plot) and the improved grid fitter (bottom plot) with cuts $Rdev < 200cm$ and $nhits \geq 30$.

for judging a fitter's performance. We applied all of four fitters currently existing in SNOMAN on a set of PMT beta gamma events (roughly 20 hours of ^{208}Tl decays on the PMT glasses) and compared their performances. Table 2 shows reconstruction efficiency of various fitters while table 3 shows their reconstruction accuracy. The quad fitter has the worst performance for the PMT beta gamma events: not only does it have a very low reconstruction efficiency, it also mis-reconstructed largest number of events into the heavy water region. The elastic fitter has similar performance as the time fitter, which is not surprising as it uses result of the time fitter as inputs. The current grid fitter is the best of all 4 fitters in SNOMAN in terms of number of mis-reconstructed events in D_2O. The improved grid fitter outperforms all fitters in SNOMAN in terms of both reconstruction efficiency and reconstruction accuracy. Figure 5 shows rfit distributions for both the time fitter and the improved grid fitter. The improved grid fitter has a much sharper peak around the PSUP position. On one hand, statistics is low and there may be difference between MC prediction and real data; on the other hand, this is results of millions of MC events and in absence of calibration sources, MC may be our best guess. Performance of the improved grid fitter is at least promising.

4.2 Real Data

Up to now, the ^{16}N source is the only calibration source which could be used to evaluate fitter performance. However, as gammas generated by the source could

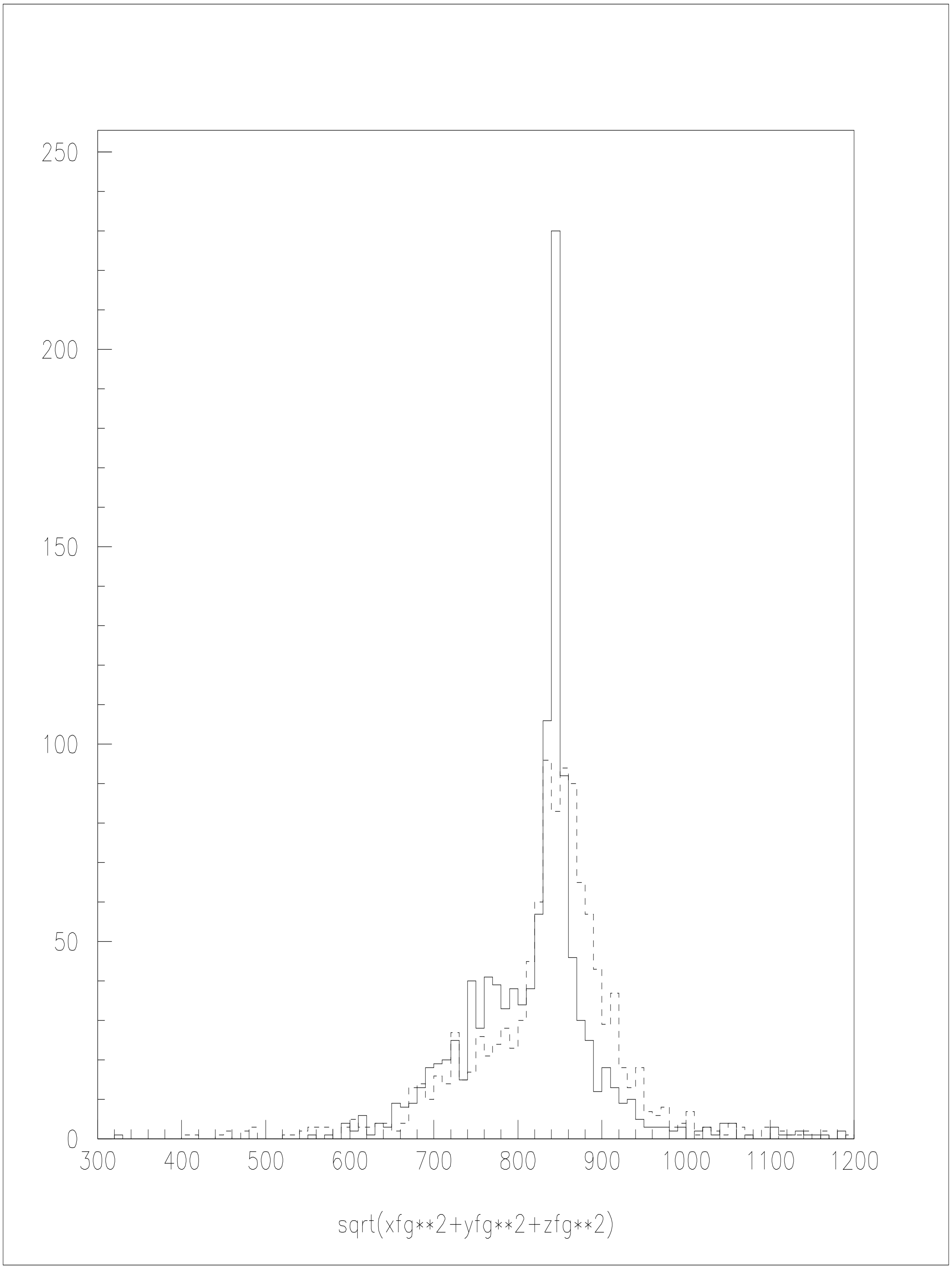

Figure 5: r_{fit} distributions of MC PMT beta gamma events reconstructed by the time fitter (dashed line) and the improved grid fitter (solid line). A cut of nhits $\geq$ 30 was imposed.

Table 2: Reconstruction efficiency of various fitters on a set of PMT beta gamma events with $nhits \geq 30$.

	O.K.	Failure
the time fitter	1168	18
the quad fitter	245	941
the elastic fitter	1158	28
orginal grid fitter	1148	38
improved grid fitter	1172	14

Table 3: Reconstruction accuracy of various fitters on a set of PMT beta gamma events with $nhits \geq 30$.

	$rfit < 600cm$	$rfit < 700cm$	$rfit < 750cm$
the time fitter	29	91	180
the quad fitter	113	217	239
the elastic fitter	30	106	192
Orginal grid fitter	20	149	304
Improved grid fitter	7	80	197

travel a long distance before being converted to electrons and starting photon production, the actual vertex is unknown and hence can't be compared with the fitted vertex on an event-by-event base. A widly used method is to calculate the angle between the event fitted direction and the vector from the source position to the event fitted position [3]. For the event reconstructed far away from the source, if the reconstructed position is correct, this angle should be small as direction of the scattered electron should be aligned as direction of the gamma which generated the electron. As shown in figure 6, there is a backward peak in the plot generated by the time fitter which indicates existance of many mis-reconstructed events. On the other hand, the improved grid fitter managed to greatly reduce the backward peak. This is another evidence that the grid fitter is better than the time fitter in terms of reducing the so called "tail" events. We also used other sets of ^{16}N data with source located at different position of the detector in the comparation of the fitters and results showed a better performance of the improved grid fitter although the improvement is not as substaintial pissibly due to minimized shadowing affect with source not located at the detector center. . Table 4 shows reconstruction effeciency on the ^{16}N events, the improved grid fitter is again better than that of the time fitter.

Another interesting thing is to look at how well fitters reconstruct non physics events, especially those non physics events which managed to leak through data cleaning cuts. Unfortunately we don't have such events sample either. If we know which event is a leaking non physics event, we may design a cut to remove such event. A set of non physics events picked up by hand scanning may be useful for such purpose. Because of time constraint, we only applied both the time fitter and

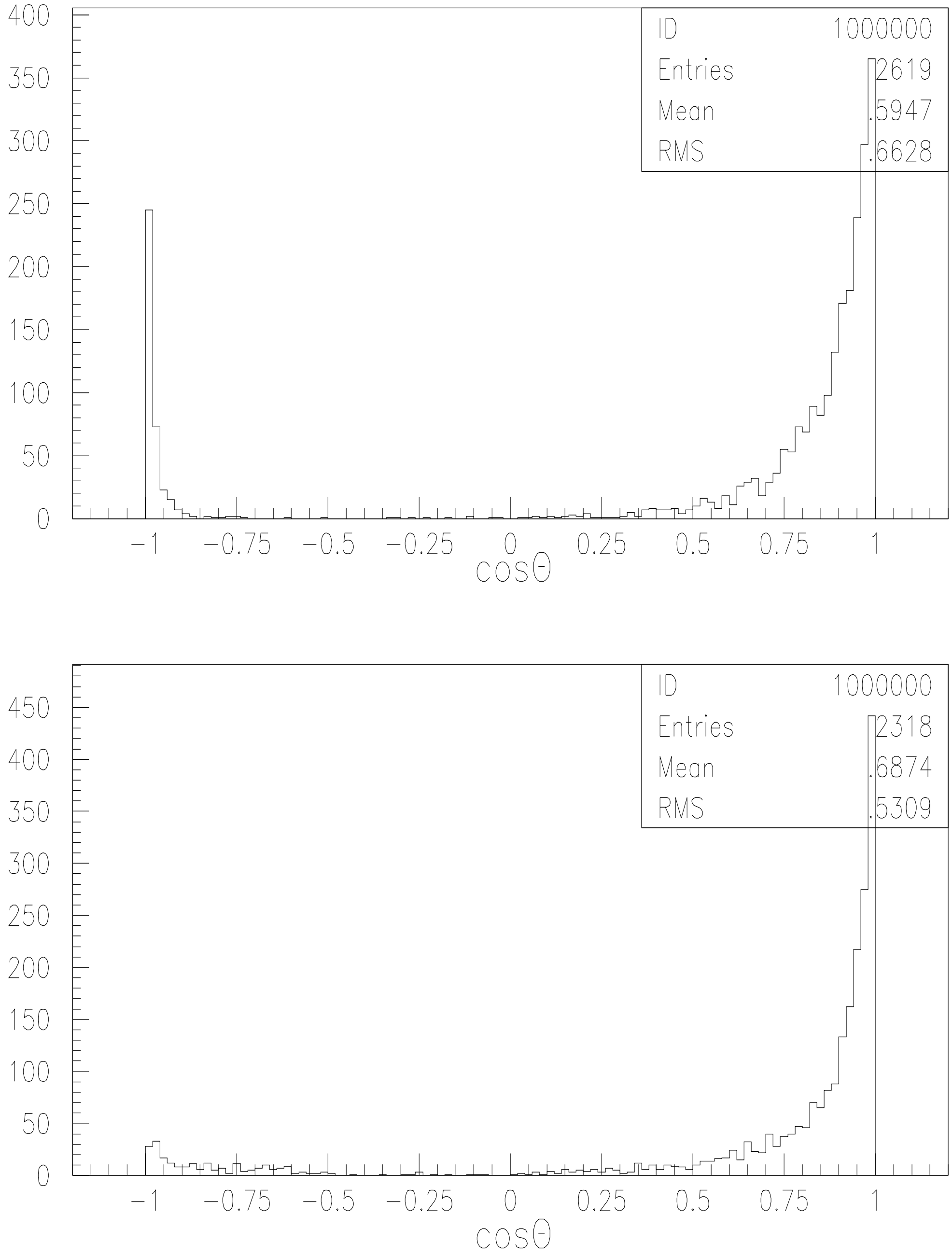

Figure 6: Cosθ distributions generated by the time fitter (upper plot) and the improved grid fitter (bottom plot). θ is the angle between the fitted direction and the vector from the source position to the fitted vertex. Data was taken during run10429 with source at the center of the detector. A cut of $nhits \geq 30$ was imposed.

Table 4: Efficiency of fitters on ^{16}N events, with a cut of $nhits \geq 30$.

	The time fitter	the improved grid fitter
OK	40730	40802
Failed	72	0

the grid fitter on the golden flasher events with results shown in figure 7. There are several points worth mentioning about the figure:

- Both fitters are poor at reconstructing the flaher events. Thus we may not be able to rely too much on fitters to remove the leaking non physics events.
- Success rate of the time fitter is much higher that that of the grid fitter on the flasher events. However, this is kind of success we don't want as the time fitter reconstructed many more events into the heavy water region. We believe main reason is that the time fitter only requires more than 10 hits left in a pre-defined time window after the reconstruction while the grid fitter requires number of hits left is more than half of the total hits. The criterion used in the grid fitter may be more reasonable as reconstruction result of an event with more than half of hits being bad is at least doubtable.
- Although it is not fair to compare two plots in figure 7 directly as two fitters have different criteria for success, the improved grid fitter seems to perform better than the time fitter on flaher events as ideally most of flasher events should be reconstructed near to the PSUP.

5 Problems and Discussions

5.1 The sliding window

In theory, the idea of applying a sliding window before reconstruction may help to remove some badly out-of-time hits. Indeed MC studies show that such technique does improve performance of the fitters although the improvement is not substantial. It was surprising to see the results when applying it to the real data, as shown in figure 8. The $r3$ distribution of events reconstructed by the grid fitter without the sliding window has a very sharp peak around the PSUP position while the time fitter also produced a much lower peak. This is similar to what is shown in figure 5. What was surprising is the r^3 distribution produced by the grid fitter with the sliding window applied before the reconstruction. The distribution is peaked in the light water region. The data were taken during run10791, run10801 and run10803. Although not 100% certain, it is much more likely that the r^3 distribuion should be peaked around the PSUP region. The events shown in figure 8 have passed the following cuts: Burst, Cluster, QvT, Pe/NHIT, Crate Isotropy, OWL, Junk, Ring of Fire, Neck, ESUM, QCluster. When we applied the sliding window before reconstructing MC events (including simulated PMT beta gamma events), the results are more or less the same as the results without the sliding window. We

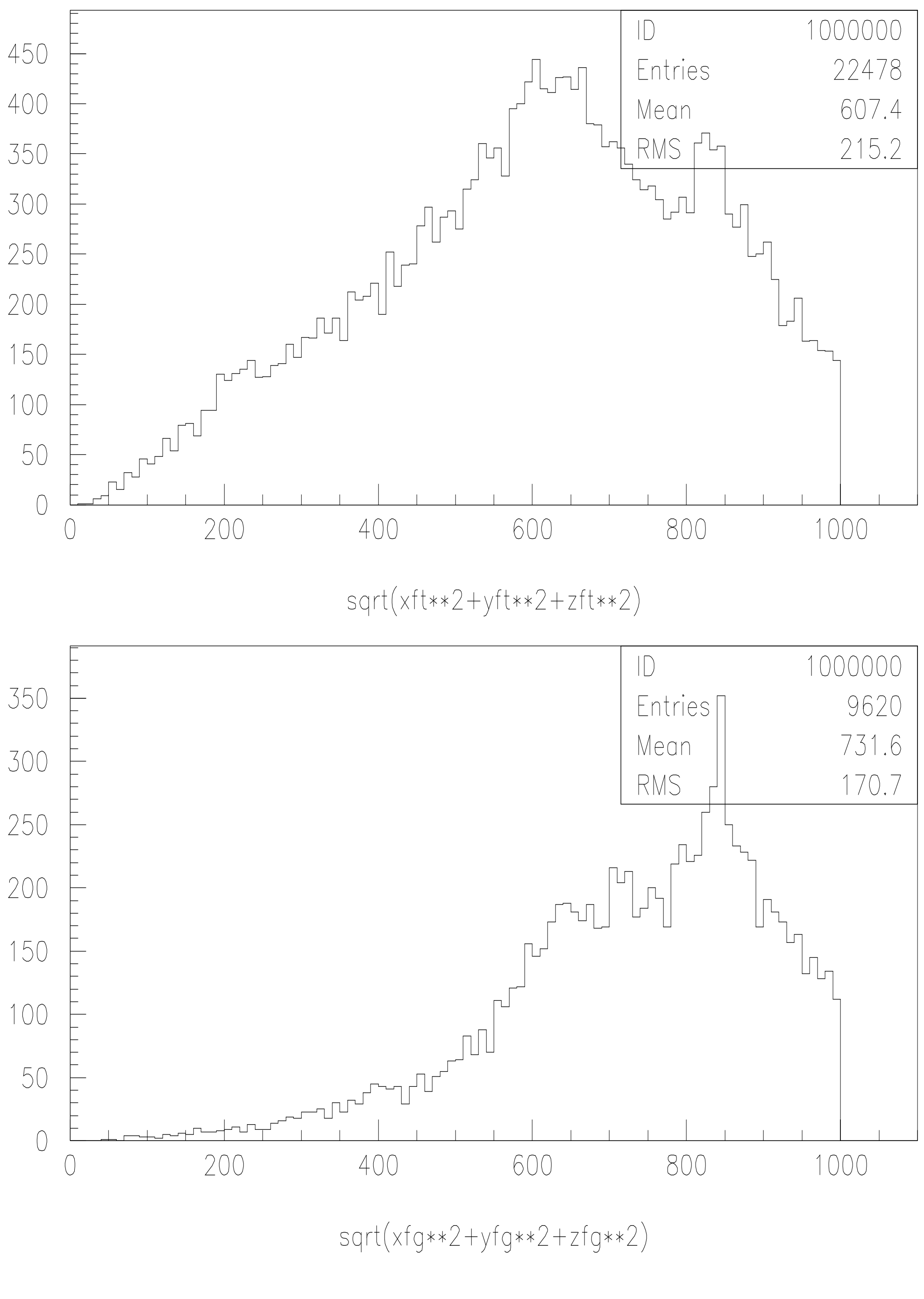

Figure 7: r_{fit} distributions of golden flasher events reconstructed by the time fitter(top plot) and the improved grid fitter(bottom plot).

Table 5: Comparision of the grid fitter with different p.d.f. on PMT beta gamma events.

	$rfit < 600cm$	$rfit < 700cm$	$rfit < 750cm$
an equally truncated gaussian	7	80	197
an asymmetrically truncated gaussian	35	205	394

also increased length of the sliding window from 76ns to 100ns, the grid fitter still produced weird r^3 distribution shown in figure 8. Two of possible reasons are:

- MC simulation of PMT beta gamma events does not agree with the reality: e.g. there may be much more scattering.
- Even after some data cleaning cuts, there are still many non physics events left which may have much wider time distribution than genuine physics events.

We are still investigating the causes. We plan to apply latest data cleaning cuts and maybe study more runs of data. At the moment, we recommand not to apply the sliding window before the reconstruction.

5.2 The probability distribution function

The probability distribution function is important for the grid fitter as it uses a maximum likelihood method. Normally different p.d.f. may produce different reconstruction results. As stated before, an asymmetrically truncated function is used as the p.d.f. for the fitter. Actually we used an equally truncated gaussian as the p.d.f. at the beginning of development of the fitter. Figure 9 shows performance of the grid fitter with an equally truncated gaussian as the p.d.f. for simulated electron events and ^{208}Tl decay events in the heavy water. Compared with figure 2 and figure 4, it seems that the equally truncated gaussian helps reconstructing events more accurately than the assymetrically truncated gaussian. Furthermore, as shown in figure 10 which can be compared with figure 6 , the grid fitter with the equally truncated gaussian as the p.d.f. managed to remove almost all of the backward peak. However, when we tried to compare reconstruction results on PMT beta gamma events with two p.d.f., performance of the equally truncated gaussian was much worse than that of the assymetrically truncated gaussian, as shown in table 5. Thus it seems different p.d.f. suits different classes of events which is really not a surprise. As improvments on fitting D_2O events with the equally truncated gaussian as the p.d.f. is not substantial and affects of PSUP events may be more serious than expected, we decided to use the assymetrically truncated gaussian as the p.d.f. When fitting real data, we may use the equally truncated gaussian as the p.d.f. **after** we are relatively certain that the events are genuine D_2O events (for example, the event is reconstructed in the D_2O region by the grid fitter with the asymmetrically truncated gaussian as the p.d.f.).

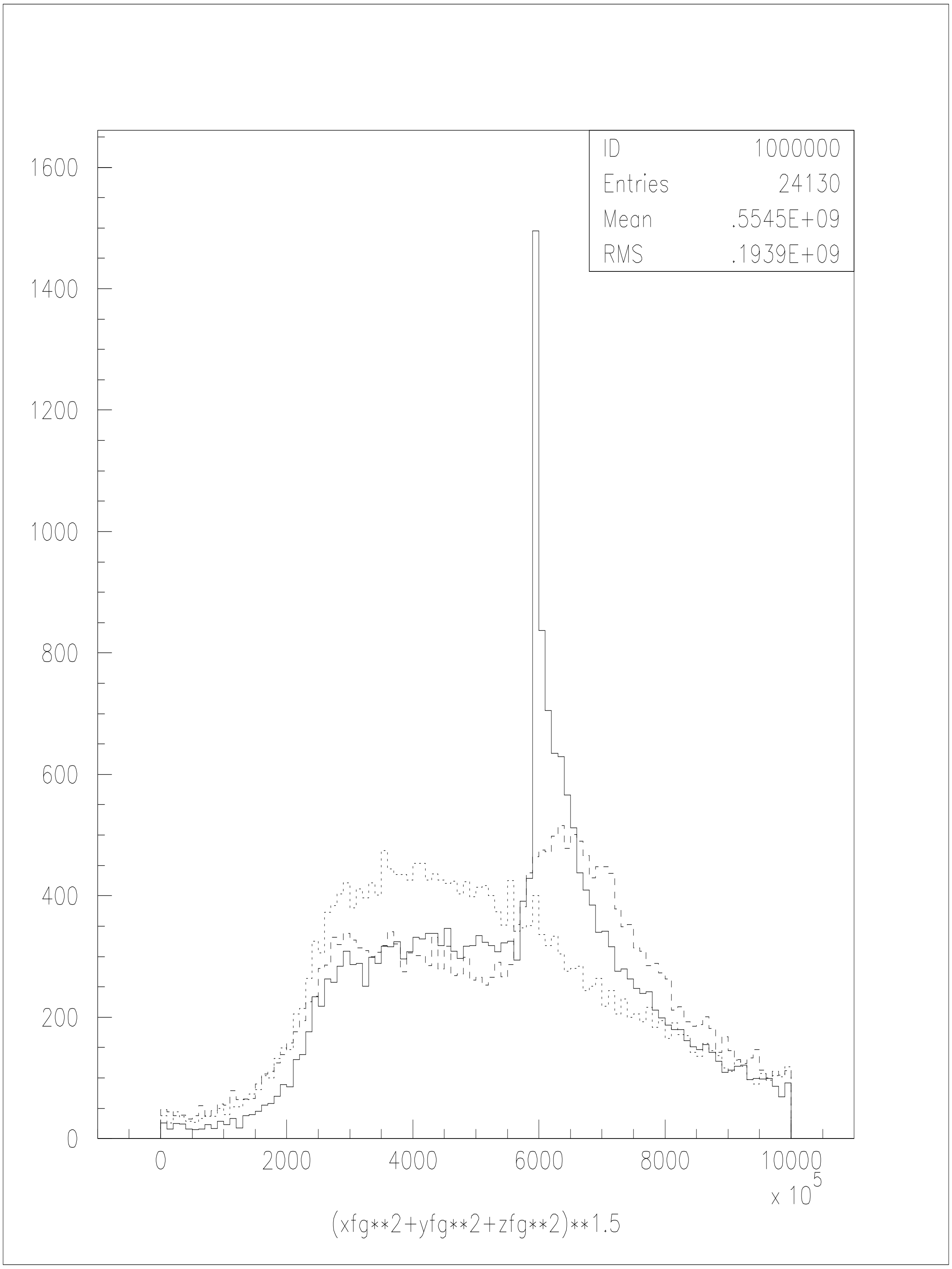

Figure 8: r^3 distributions of events reconstructed by the grid fitter without the sliding window(solid line), the grid fitter with the sliding window(dotted line) and the time fitter(dashed line).

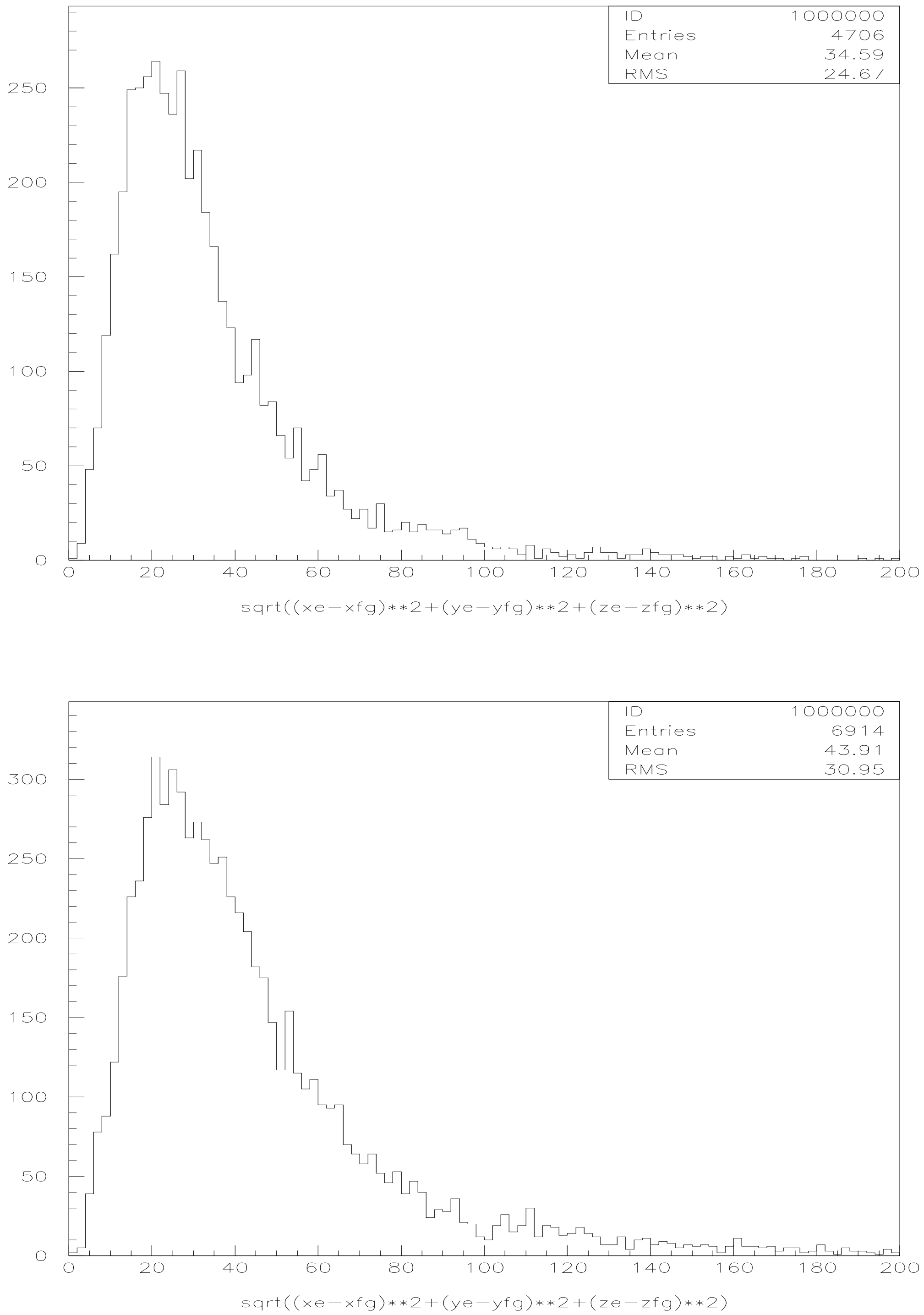

Figure 9: R_{dev} distributions for MC simulated 5Mev electron events (upper plot) and ^{208}Tl decay events in D_2O (bottom plot) reconstructed by the grid fitter using an equally truncated gaussian as the p.d.f..

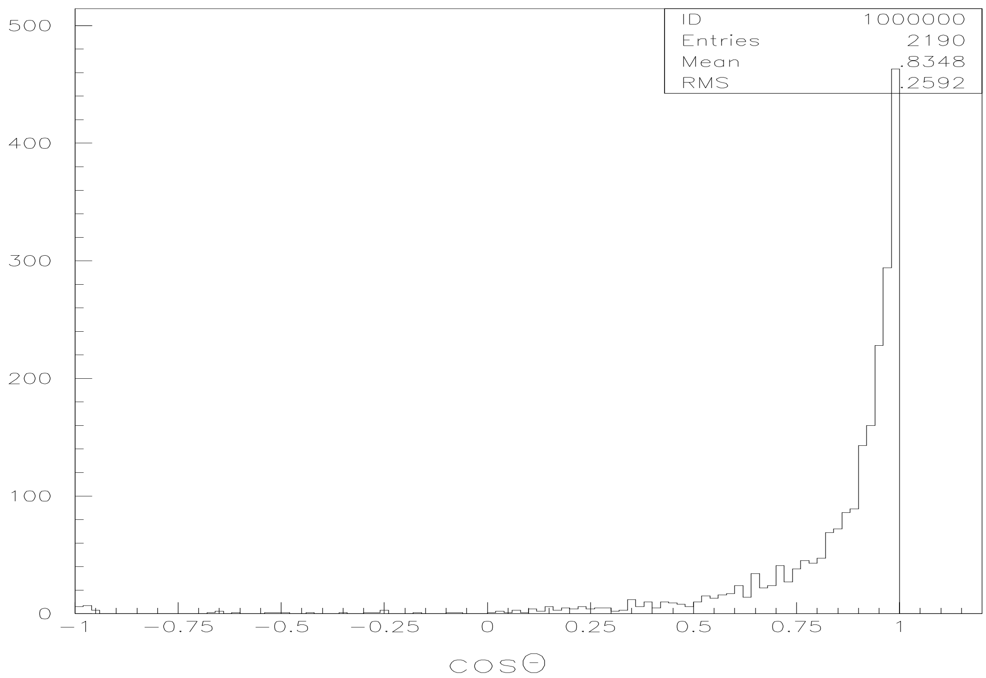

Figure 10: Cosθ distributions generated by the grid fitter using an equally truncated gaussian as the p.d.f.. θ is the angle between the fitted direction and the vector from the source position to the fitted vertex. Data was taken during run10429 with source at the center of the detector. A cut of $nhits \geq 30$ was imposed.

6 Conclusion and Comments

We developed a new fitter based on the grid fitter in SNOMAN. Tests on both MC data and real data showed that the improved grid fitter is able to reconstruct event vertices with higher efficiency and smaller tails in Rdev than the time fitter. In particular, this improved grid fitter works better than all existing fitters in SNOMAN in reconstructing PMT beta gamma events.

As the ^{16}N source is a gamma source, evaluations of the fitters based on its data are indirect and may be lack of definiteness. It is necessary to have other calibration sources such as the ^{8}Li source to make more conclusive statement..

One possible way to enhance performance of the fitter is to use value of its chi sqaure or likelihood function to judge quality of the reconstruction. Some of our initial attempts have not produced tangible successes.

References

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