HOWTO use the SNO Salt Flux Results

SNO Collaboration

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1 Introduction

This document provides technical details about the data presented in SNO's salt flux publication **Measurement of the Total Active ^{8}B Solar Neutrino Flux at the Sudbury Neutrino Observatory with Enhanced Neutral Current Sensitivity** [1]. It outlines how SNO used the data in this publication to produce its own oscillation contour plots and describes how to include the required correlations for readers interested in including the new salt data in similar oscillation analyses.

2 Differences with the Previous SNO Analysis

2.1 Pure D_2O Data Approach

The CC, ES and NC "fluxes" presented in the earlier SNO papers that analyzed data from the pure D_2O phase [2], [3], were in general derived under the explicit assumption of an undistorted ^{8}B spectral shape (i.e. an energy-independent survival probability), except where explicitly stated otherwise.

The assumption of a ^{8}B spectrum was appropriate for testing the null hypothesis of neutrino oscillations. (We assumed no oscillations, then showed that our data under this assumption would imply a non-zero flux of $\nu_{\mu,\tau}$, in contradiction, thus refuting the null hypothesis.) However using fluxes derived assuming an undistorted spectrum is not appropriate for calculating constraints on MSW mixing parameters, since MSW oscillations generally allow for spectral distortions. We did not use the reported integral fluxes when making the contour plots in [3], but instead used the summed energy spectra. During the pure D_2O phase, the spectral shape was particularly important for distinguishing CC from NC events, since our best handle on detecting neutrons was to look for a "bump" in the energy spectrum, due to the neutrons.

For these reasons, SNO used a spectrum-based approach [4] in the oscillation analysis of data from the pure D_2O phase. This approach was sensitive to the CC and NC flux information embodied in the energy spectrum. At each grid point in the Δm^2–$\tan^2\theta$ plane, we calculated the expected summed energy spectra (CC+NC+ES+bkgd events, day and night), and compared the calculated energy spectra to the SNO data.

2.2 Salt Phase Data Approach

With the salt phase data, SNO is using a different approach for generating contour plots. Because of the different event isotropy distributions for neutron capture events compared to CC or ES events, we can effectively separate NC events from other kinds of events using radial, isotropy, and angular information. We do not need to make assumptions about the shapes of the CC or ES energy spectra when deriving these fluxes. SNO determines the salt fluxes independent of assumptions about the energy dependence of the ν_e survival probability. These fluxes are therefore appropriate for use in physics interpretation and an MSW oscillation analysis. (The salt paper also includes, for comparison with previous analyses, fluxes derived under the assumption of an undistorted ^{8}B energy spectrum. These values are *not* appropriate for drawing contour plots, for the reasons given above.)

When including the salt data in our new oscillation analysis, we used *only* the integral CC, ES and NC fluxes. We did not include spectral information such as is shown in Figure 2c of the paper. There are multiple reasons for this:

- For the salt data, the NC peak is much larger, broader, and higher in energy. The energy spectrum of neutron capture events is more similar to the energy spectrum of CC events than it was for the pure D_2O data. Therefore, the energy spectrum alone does not provide as much useful separation between CC and NC events as it did in pure D_2O.
- Most of SNO's exclusion power in the oscillation analysis comes from the CC and NC fluxes (effectively from the CC/NC ratio, a direct measure of the survival probability). From the salt data, we obtain

better constraints on the CC/NC ratio from the CC, NC and ES "unconstrained" fluxes than we do if we just use information from the summed energy spectrum. This is because the isotropy parameter β_{14} is very useful for distinguishing CC and NC events in the salt data.

- The analysis using integral fluxes is much simpler than that using the energy spectrum. This was not an option for the pure D_2O data, where we relied upon the shape of the energy spectrum to determine the CC and NC contributions. Since the salt flux data are free of spectral assumptions, an oscillation analysis with them is possible.

- SNO completed an analysis using the salt spectrum (and not the fluxes) to compare with the analysis presented in the paper with only salt fluxes. We found that the constraints coming from precise CC and NC fluxes are stronger than those coming from spectral shape (given the current size of the errors bars in the energy spectrum).

Information is discarded when the shape of the energy spectrum is not used. But with SNO's salt data, we gain more by using the precise fluxes (which make use of the statistical separation from the isotropy parameter β_{14}) and throwing away the energy spectrum, than we do by including the energy spectrum (and hence throwing away the isotropy information). A more advanced MSW oscillation analysis is being planned for the future that will jointly use the isotropy and energy spectrum information in a maximum-likelihood fit.

At this point, we do not recommend trying to combine the integral flux data with the spectral data in Figure 2c of the salt paper. There is the obvious risk that one must avoid fitting the same data set twice; though this can be done properly, some care is required. More importantly, there are many correlations between flux uncertainties and spectral uncertainties in such an approach that need to be taken into account. In addition to statistical correlations there are experimental correlations between parameters in the data (e.g. energy and β_{14}) that are significant and not simply described in short order. These will be made available along with SNO's more advanced spectral analysis, at a later date.

3 Caution when Considering the CC/NC Ratio

The ratio of the fluxes, CC/NC, could potentially be used for physics interpretation. The ratio is a direct measure of the survival probability for electron neutrinos. That, plus the fact that some systematic uncertainties cancel in the CC/NC ratio make it appealing. However, the reader should be aware that the uncertainties in the ratio are not Gaussian distributed. In our paper [1], we gave the *equivalent* 1σ uncertainty in this ratio; this σ is not to be used with a normal probability distribution.

4 Statistical Correlations

We draw attention to the fact that our CC, ES and NC fluxes are *statistically* correlated, since they are derived from a fit to a single data set with imperfect statistical separation between different kinds of events. The statistical correlation coefficients between the fluxes, coming from our maximum likelihood analysis, are:

$$\rho_{CC,NC} = -0.521 \quad (1)$$
$$\rho_{CC,ES} = -0.156 \quad (2)$$
$$\rho_{ES,NC} = -0.064 \quad (3)$$

These can be used with the statistical uncertainties from the paper to write down the statistical covariance matrix for the salt fluxes:

$$\mathbf{V_{stat}} = \begin{pmatrix} (0.27)^2 & (-0.521)(0.074)(0.27) & (-0.064)(0.27)(0.29) \\ (-0.521)(0.074)(0.27) & (0.074)^2 & (-0.156)(0.074)(0.29) \\ (-0.064)(0.27)(0.29) & (-0.156)(0.074)(0.29) & (0.29)^2 \end{pmatrix}$$

(Here the statistical error bars on the fluxes have been symmetrized.) The rows (columns) correspond to the NC, CC and ES flux uncertainties.

Systematic	NC	CC	ES
energy scale	+1	+1	+1
energy resolution	+1	+1	+1
energy non-linearity	+1	+1	+1
radial accuracy	+1	+1	+1
vertex resolution	+1	+1	+1
angular resolution	+1	+1	-1
isotropy mean	+1	-1	-1
isotropy resolution	+1	+1	+1
radial energy bias	+1	+1	+1
vertex X accuracy	+1	+1	+1
vertex Y accuracy	+1	+1	+1
vertex Z accuracy	+1	-1	-1
internal neutron background	+1	0	0
internal background $\gamma's$	+1	+1	+1
neutron capture	+1	0	0
Cherenkov backgrounds	+1	+1	+1
"AV events"	+1	+1	+1

Table 1: Signs of systematic correlations, relative to its effect on the NC flux. An entry of +1 indicates a 100% positive correlation, −1 a 100% negative correlation, and 0 means no correlation.

5 Systematic Correlations Between Fluxes

Systematic uncertainties between fluxes can be correlated as well. Some sources of systematic uncertainty, such as neutron capture uncertainty, affect only one of the three fluxes, and so can be considered to be uncorrelated with the other fluxes. The systematic covariance matrix for neutron capture efficiency thus looks like:

$$\mathbf{V_{neut.cap.}} = \begin{pmatrix} (0.135)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(The uncertainty here in this matrix is just the 2.6% neutron capture systematic times the central value of the NC flux.)

Other systematics can be either 100% correlated (e.g. radial accuracy) or 100% anticorrelated (e.g. isotropy mean). For example, a change in the fiducial volume coming from a reconstruction radial scaling error (the "radial accuracy" systematic) has the same sign for all three fluxes. So using the radial accuracy systematic uncertainties from Table II of the paper, we get:

$$\mathbf{V_{radialacc.}} = \begin{pmatrix} (0.169)^2 & +(0.169)(0.0405) & +(0.169)(0.0608) \\ +(0.169)(0.0405) & (0.0405)^2 & +(0.0405)(0.0608) \\ +(0.169)(0.0608) & +(0.0405)(0.0608) & (0.0608)^2 \end{pmatrix}$$

The most important anticorrelated systematic is the "isotropy mean". Isotropy is important for separating CC and ES events from NC events, so CC and ES will have a negative correlation with the NC flux (and a postive correlation with each other) for this systematic:

$$\mathbf{V_{isotropymean}} = \begin{pmatrix} (0.169)^2 & -(0.169)(0.0477) & -(0.169)(0.0221) \\ -(0.169)(0.0477) & (0.0477)^2 & +(0.0477)(0.0221) \\ -(0.169)(0.0221) & +(0.0477)(0.221) & (0.0221)^2 \end{pmatrix}$$

Table 1 shows the sign of the correlation for each systematic in Table II of SNO's salt paper.

Using the table of systematics (Table II) from the paper and the signs for the correlations, one can assemble an individual covariance matrix for each systematic. Then, to get the *total* covariance matrix for the CC, ES and NC fluxes, one simply adds all of the covariance matrices together:

$$\mathbf{V_{total}} = \mathbf{V_{stat}} + \mathbf{V_{sys,1}} + \mathbf{V_{sys,2}} + \mathbf{V_{sys,3}} \cdots \tag{4}$$

Even when fluxes are being analyzed as opposed to energy spectra, it is best to determine the effect of energy-related systematics at each grid point in the Δm^2–$\tan^2\theta$ plane. For the salt analysis, these would include energy scale and energy resolution; energy non-linearity is tiny and its uncertainty is small enough that it can reasonably be ignored. For all other systematics, it is safe to assume that their effect on the rates (flat in the spectrum) is the same for all oscillation parameters.

6 Energy Scale and Resolution Uncertainties

When determining the effect of energy-related systematics at each grid point, energy scale and resolution uncertainties are applied to the theoretical spectrum. During the salt phase in SNO, the energy scale uncertainty should be implemented as a $\pm 1.1\%$ gain shift in total energy. Energy resolution has an uncertainty which is energy dependent, and is described by the function:

$$\frac{\Delta\sigma_T}{\sigma_T} = \begin{array}{ll} 0.035 + 0.00471 \times (T - 4.975), & \text{if } T > 4.975 \text{ MeV} \\ 0.035 & \text{if } T < 4.975 \text{ MeV}. \end{array} \tag{5}$$

7 Salt Fluxes and χ^2

When the "CC flux" is quoted by SNO as: 1.59×10^6 cm^{-2} s^{-1}, that's saying that the number of events SNO observed in the salt data set, above kinetic energy 5.5 MeV, attributed to CC interactions in the fit, is equal to the number of CC events that would be observed, above kinetic energy 5.5 MeV, if the integral flux (from zero to endpoint) of ν_e had the value of 1.59×10^6 cm^{-2} s^{-1} and had the ^{8}B spectral shape. The ^{8}B spectral shape aspect of this definition is only for normalization; there is no assumption of any spectral shape when extracting the number of events in SNO's salt phase. Similarly, for the NC and ES fluxes.

When calculating the theoretical CC flux for a set of oscillation parameters, for comparison with SNO data, one should be aware of the above definition. Thus, one should not be calculating,

$$\text{don't calculate the total } \nu_e \text{ flux: } f_B \int_0^\infty \phi_{SSM}(E_\nu) P_{ee}(E_\nu)\, dE_\nu \tag{6}$$

but rather,

$$\text{use: } f_B \int_0^\infty \phi_{SSM}(E_\nu)\, dE_\nu \; \frac{\int_0^\infty \int_0^\infty \int_{5.5}^\infty \phi_{SSM}(E_\nu) P_{ee}(E_\nu) \frac{d\sigma}{dT_e}(E_\nu, T_e) R(T_e, T)\, dE_\nu\, dT_e\, dT}{\int_0^\infty \int_0^\infty \int_{5.5}^\infty \phi_{SSM}(E_\nu) \frac{d\sigma}{dT_e}(E_\nu, T_e) R(T_e, T)\, dE_\nu\, dT_e\, dT} \tag{7}$$

where f_B is a factor which allows one to float the total ^{8}B solar neutrino flux from the SSM value, P_{ee} is the survival probability for a ν_e produced in the Sun to be detected as a ν_e, and $R(T_e, T)$ is the energy response function,

$$R(T_e, T) = \frac{1}{\sqrt{2\pi}\sigma_T} \exp\left[-\frac{(T_e - T)^2}{2\sigma_T^2}\right] \tag{8}$$

and T_e is the true recoil electron kinetic energy and T is the observed electron kinetic energy, with resolution,

$$\sigma_T(T) = -0.145 + 0.392\sqrt{T} + 0.0353\, T. \tag{9}$$

It's a similar definition for SNO's ES flux, remembering to include the contribution from $\nu_{\mu,\tau}$ using the appropriate cross section for $\nu_{\mu,\tau}$, and $1 - P_{ee}(E_\nu)$. There is no ambiguity in interpreting SNO's NC flux data. It's easy to see that procedurally, SNO's NC event extraction is being interpreted as, and is equal to, a flux:

$$\text{SNO NC flux is: } f_B \int_0^\infty \phi_{SSM}(E_\nu)\, dE_\nu \tag{10}$$

For an oscillation analysis, comparing predicted fluxes as defined above with SNO measured fluxes, χ^2 is easy to calculate. We define a vector:

$$\vec{a}(\phi_B) = (NC_{meas} - NC_{predicted}, CC_{meas} - CC_{predicted}, ES_{meas} - ES_{predicted}).$$

Then the χ^2 is given by:

$$\chi^2(\phi_B) = \vec{a}^T \cdot (\mathbf{V_{total}})^{-1} \cdot \vec{a} \tag{11}$$

Minimizing the χ^2 with respect to f_B at each point, we then draw χ^2 contours in the usual way.

When calculating the effect of energy-related systematics at each grid point in the Δm^2–$\tan^2\theta$ plane, it is necessary to "detach" those systematics from the SNO data uncertainties, and apply them to the model with uncertainties described in Section 6, rather than to the data. For other systematics, SNO considers it acceptable to apply the uncertainties to the observed rates, as given in [1].

8 Correlations with Other Experiments

The χ^2 calculated above from the NC, CC and ES fluxes can then be added to a global analysis. Correlated systematics with other experiments, such as cross section uncertainties or uncertainties in the ^{8}B shape can then be accounted for in the usual way by including covariance terms between different experimental results. The effect of the systematic uncertainty in the ^{8}B spectral shape is best determined at each grid point in the Δm^2–$\tan^2\theta$ plane.

Please note that in SNO's oscillation analysis presented in [1], while we used only integral fluxes from the salt phase, we still included day and night energy spectra from our D_2O phase in the global fit, as discussed in Section 2.1. That is, while we did a flux-only analysis for the salt data, we have not changed our basic methodology for handling the D_2O phase data [4].

We would also like to comment on experimental correlations between SNO's salt results and its D_2O phase results. There are certainly correlations, since it's the same detector. However, these correlations are not as large as one might expect, since some changes have been made to both the detector and the analysis procedures. For example, SNO used different reconstruction algorithms for the two data sets, and for that and other reasons we expect there to be little correlation between reconstruction-related uncertainties between the two data sets. Similarly, changes in optics and in the Monte Carlo model (e.g. time-varying gain drifts) reduce correlations in the energy scale uncertainty. SNO has tested the sensitivity of its global fits to assumptions about the experimental correlations between the D_2O and salt data sets, and finds that these effects are generally quite small. Because proper treatment of experimental correlations requires very detailed knowledge of the SNO detector, and since these correlations have little practical impact on the resulting contours, we recommend that others wishing to fit SNO data ignore experimental correlations between SNO's data sets for the time being, as a reasonable approximation, despite the fact that SNO does include its best assessment of these correlations in our own oscillation analysis.

9 Contact Persons

For more information about how SNO produces MSW contours from its data, you may contact the following individuals, who will forward your questions to the appropriate parties:

- Mark Chen (mchen@post.queensu.ca)
- Yasuo Takeuchi (takeuchi@owl.phy.queensu.ca)
- Scott Oser (oser@physics.ubc.ca)

References

[1] *Measurement of the Total Active ^{8}B Solar Neutrino Flux at the Sudbury Neutrino Observatory with Enhanced Neutral Current Sensitivity* , S.N. Ahmed *et al.*, `submitted to Phys. Rev. Lett.`

[2] *Direct Evidence for Neutrino Flavor Transformation from Neutral-Current Interactions in the Sudbury Neutrino Observatory* , Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002).

[3] *Measurement of Day and Night Neutrino Energy Spectra at SNO and Constraints on Neutrino Mixing Parameters*, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011302 (2002).

[4] For details see *HOWTO use the SNO Solar Neutrino Spectral Data*, available at http://sno.phy.queensu.ca.