Updated νd and $\overline{\nu}$d Cross Sections in the MSW Code in SNOMAN

SNO-STR-2002-007

I. Lawson
University of Guelph
September 16, 2002

Abstract

Recently, new CC and NC total and differential cross sections were published by the Kubodera group. These cross sections have been added into the MSW code and this note shows their consistency with the previous results. In addition, the coupling constant for the Effective Field Theory method is re-examined for the new cross sections. Finally, in an effort to help streamline the anti-neutrino studies, the MSW code was re-configured slightly to enable it to generate anti-neutrino events. The output is described and the methods to generate these events are discussed.

1 Cross Sections

Three different solar neutrino interactions can occur within the SNO detector: electron scattering (ES), charged current (CC) and neutral current (NC). The reactions are

$$\begin{array}{lcll} \nu_x + \mathrm{e}^- & \longrightarrow & \nu_x' + e^{-'} & \mathrm{ES} \\ \nu_e + \mathrm{d} & \longrightarrow & p + p + e^- & \mathrm{CC} \\ \nu_x + \mathrm{d} & \longrightarrow & p + n + \nu_x' & \mathrm{NC}. \end{array}$$

The last two reactions only occur when the threshold energy exceeds a minimum value of 1.442 MeV for CC and 2.224 MeV for NC reactions. The SNOMAN MSW code uses the total cross sections to calculate the daily rates of each interaction while the differential cross sections are used when the actual Monte Carlo is run to predict the energy and direction of the electron for CC and ES interactions and the neutron for NC interactions.

In addition to the reactions described above, anti-neutrino reactions may also be possible within the SNO detector. The reactions of interest are:

$$\begin{array}{lcll} \overline{\nu_e} + \mathrm{d} & \longrightarrow & n + n + e^+ & \overline{\mathrm{CC}} \\ \overline{\nu_x} + \mathrm{d} & \longrightarrow & p + n + \overline{\nu_x}' & \overline{\mathrm{NC}}. \end{array}$$

The anti-NC reaction is similar the NC reaction; in SNO these two reactions probably cannot be distinguished since the observable particles in the final state are the same. On the other had, the anti-CC is clearly different. SNO has the capability of observing the e^+ and two neutrons (or a subset of these particles). The threshold energy for the anti-CC reaction is higher at about 4.029 MeV. The number of events can be predicted by the MSW code in the same manner as for regular CC events with some tweaking of the Fortran routines. These changes will be outlined and the results described showing that the cross sections agree well with expectations.

1.1 Neutrino-Deuteron Interactions

1.1.1 Total Cross Sections

The total cross sections for the CC and NC interactions are taken from tables provided to SNO by various Theorists. The cross sections are calculated analytically at discrete energy values that are relevant for SNO. In SNOMAN before version 4.00183, only two sets of cross sections were implemented, these were the K. Kubodera and S. Nozawa (KN) [2] and and S. Ying, W.C. Haxton and E.M. Henley (YHH) [3] cross sections. Newer versions of SNOMAN have an updated set of tables containing cross sections from Kubodera *et al.* (NSGK) [4], Butler, Chen and Kong (BCK) [5] and Ellis, Bahcall and Lisi (EBL) [6]. Note that EBL calculated only the CC cross sections. Recently, Nakamura *et al.*(NETAL) [7] published new cross section values for CC, NC, anti-CC and anti-NC reactions. These cross sections are about 1% larger than the previously reported values and they give the conclusion that it is reasonable to assign a 1% systematic uncertainty to the νd cross sections. The new results do take into account some of the radiative corrections (RC) discussed in various papers, such as Kurylov *et al.* [8]. They conclude that the CC cross section may increase by about 2% if the remaining RC are included while the NC reaction is expected to lie within the quoted ∼1% error quoted in their paper. Thus the new NETAL CC and NC reactions probably should not be used with the radiative correction routines included in SNOMAN by J. Formagio [9], but can be used for systematic studies if one so desires.

I will now give a brief outline as to what happens in the MSW code to generate ones desired events. The NSGK group calculated the cross sections using the phenomenological Lagrangian approach using new and more accurate models that became available since their first publication. The BCK group use an Effective Field Theory approach to calculate the cross sections which are dependent on a floating parameter known as the coupling constant of the axial two-body current, $L_{1,A}$, in units of fm^3. The default value of $L_{1,A}$ in SNOMAN is 5.6, since this is the value that agrees best with the NSGK total cross section values. The formal theoretical uncertainty in the BCK calculation is taken to be 3%, for more information on how BCK determine this the reader should check out their paper [5]. H. Robertson [10] confirms the estimated for $L_{1,A}$. For the new NETAL results a different $L_{1,A}$ value should be used and this will be discussed below.

The cross sections, differential cross sections and energy distributions for the various reactions will now be described and verified for consistency. For CC and NC reactions only the recent NSGK, BCK and NETAL results will be shown, while for the anti-CC and NC reactions the KN and NETAL results will be shown.

$L_{1,A}$ Consistency Check

Previous studies by Robertson [10] showed that the BCK cross sections agreed within 0.5% with the NSGK cross sections for the CC reactions when the value of $L_{1,A}$ was chosen to be 5.6 fm^3, figure 1 verifies those conclusions to be correct and also shows that the NC reactions also agree within 0.5%.

A similar study can be done using the NETAL cross sections. First the best value of L_{1A} must be found for the BCK formalism. The value of $L_{1A} = 6.2$ fm^3 was found to be the best value. This value also agrees within 0.5% with the calculations done by NETAL for both the CC and NC reactions. Note that the NETAL results do take into account some of the radiative corrections, as described in their paper and this causes the NETAL result to differ from the NSGK and BCK results with respect to neutrino energy, these differences will be shown below.

Charged Current Cross Sections

Figure 3 shows the CC cross sections for the NSGK, BCK and NETAL reactions, the value of L_{1A} was 5.6 fm^3 and 6.2 fm^3. The logarithm of the cross sections is also plotted to observe any large differences at low neutrino energies.

Figure 4 shows the difference between the NSGK, BCK and NETAL cross sections. Several features can be seen at low neutrino energies The kink for the NSGK cross sections at 2 MeV suggests that the interpolation to low energies may have some slight problems. The BCK and NSGK cross sections agree at the level of 1% from 2.5 MeV while the NETAL cross section agrees within 1% of the NSGK and BCK cross sections with L_{1A} equal to 5.6 or 6.2 fm^3.

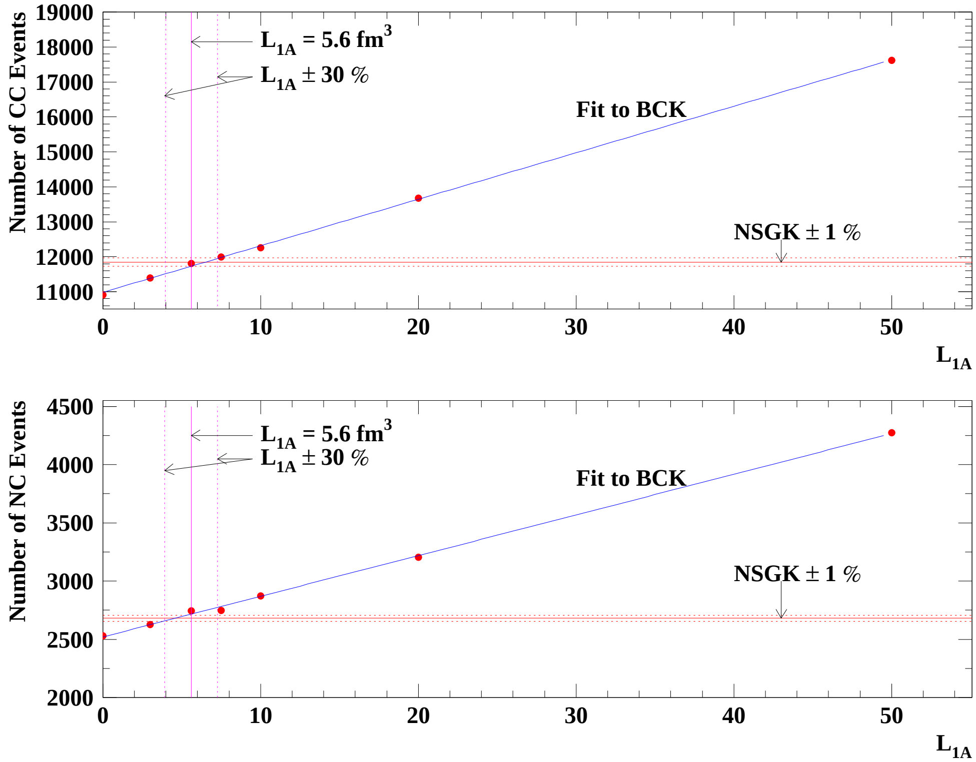

Figure 1: The CC and NC reaction rates as a function of L_{1A}, compared to the NSGK rate.

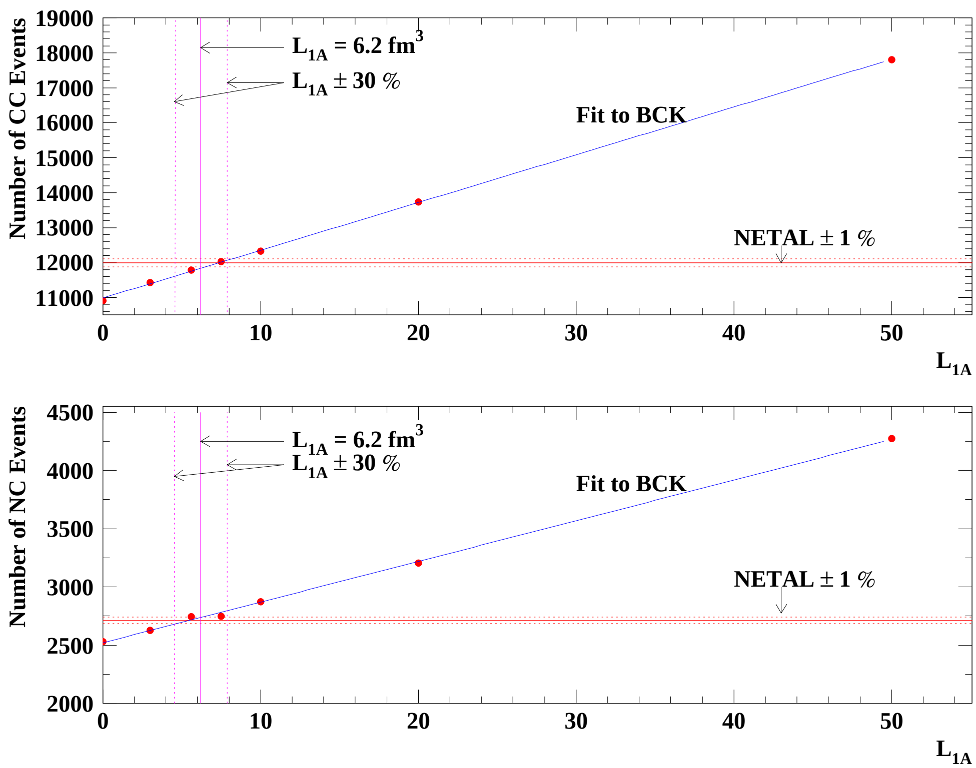

Figure 2: The CC and NC reaction rates as a function of L_{1A}, compared to the NETAL rate. The value of L_{1A} at 5.6 fm^3 agrees with NETAL within 30%, the best value is about 6.2 fm^3.

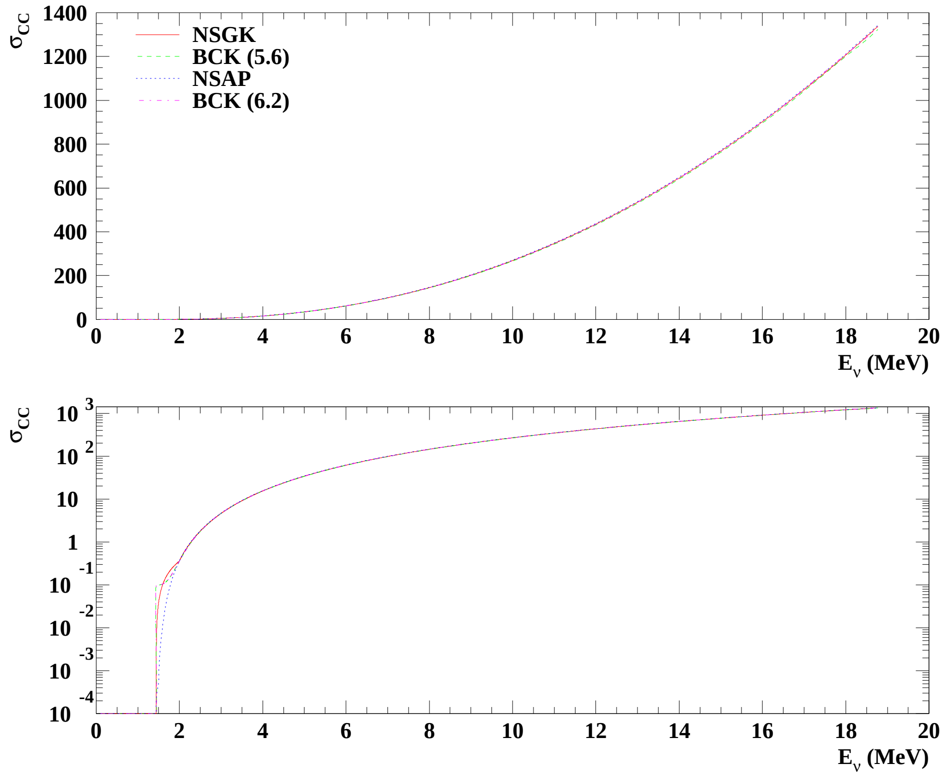

Figure 3: The charged current total cross section with respect to neutrino energy (E_{nu}) for the NSGK, BCK and NETAL cross section calculations, where L_{1A} is 5.6 and 6.2 fm^3.

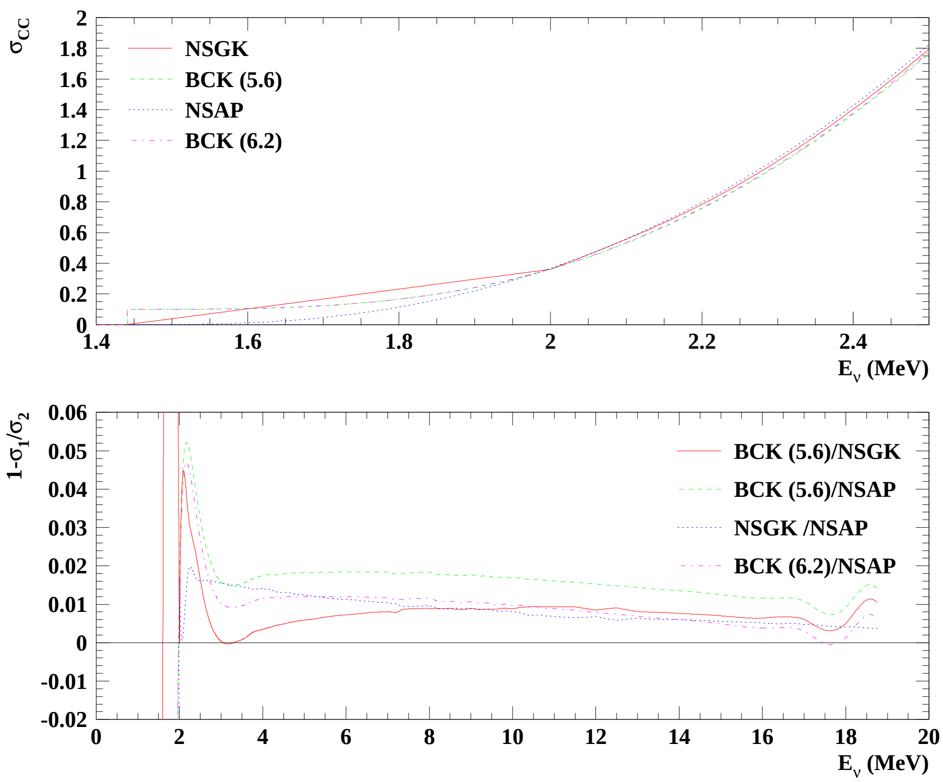

Figure 4: The top plot shows a zoom in on the charged current total cross section at neutrino energies (E_{nu}) between 1.4 and 2.5 MeV for the different cross section calculations. The bottom plot shows (1.0-ratios) of the BCK, NSGK and NETAL cross sections.

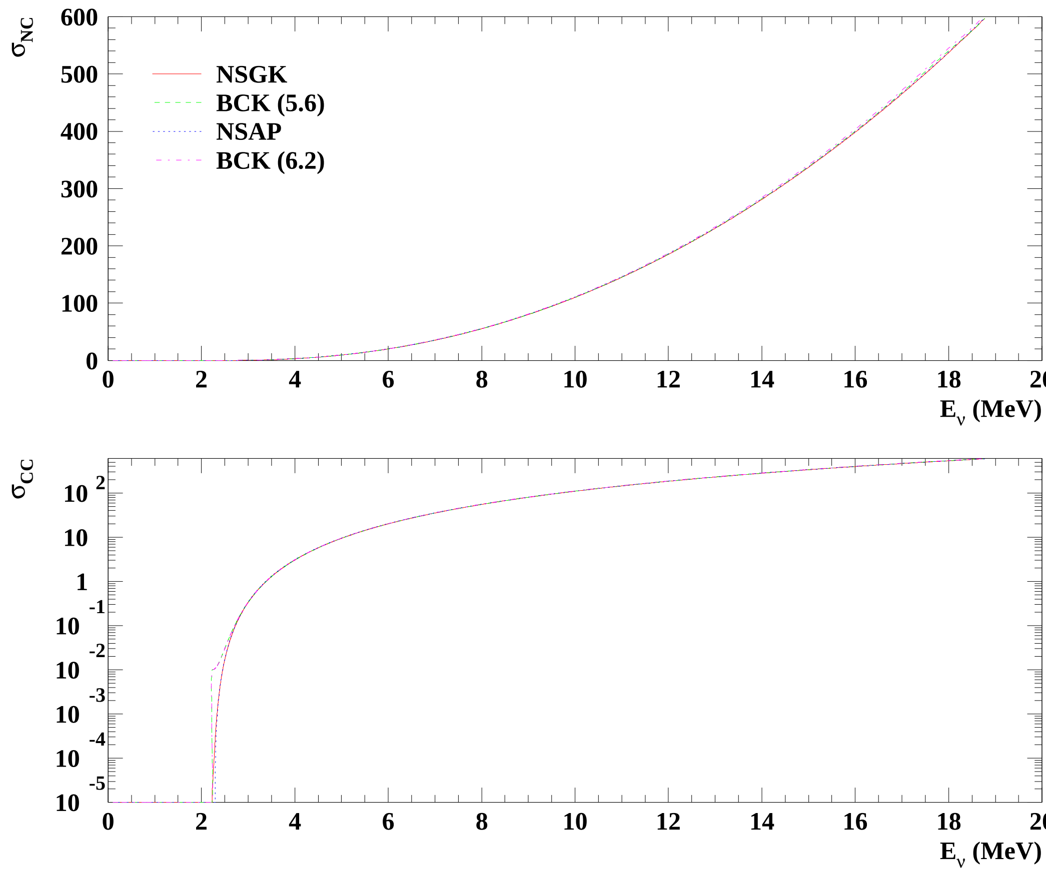

Figure 5: The neutral current total cross sections with respect to neutrino energy (E_{nu}) for the different cross section calculations.

Neutral Current Cross Sections

Figure 5 shows the NC cross sections for the NSGK, BCK and NETAL cross sections in SNOMAN. The logarithm of the cross sections is also plotted to observe any large differences at low neutrino energies.

Figure 6 shows the difference between the NSGK, BCK and NETAL cross sections. At low energies there are some differences between the different cross sections which depend on the interpolation to cross sections below the lowest tabulated points. The NETAL, BCK and NSGK cross sections agree at the level 0.5% from neutrino energies greater than 3.0 MeV, when one compares the NETAL cross section with BCK where L_{1A} is 6.2 fm^3 and NSGK with BCK where $L_{1A} = 5.6$ fm^3.

Anti-Charged Current Cross Sections

Figure 7 shows the anti-CC cross sections for the KN and NETAL reaction. The lower plot shows the difference between the KN and NETAL cross sections. The KN and NETAL cross sections agree at better than 1% from 8 MeV. Between 4 MeV and 8 MeV, the agreement is not as good, since it is energy dependent. This difference is probably due to

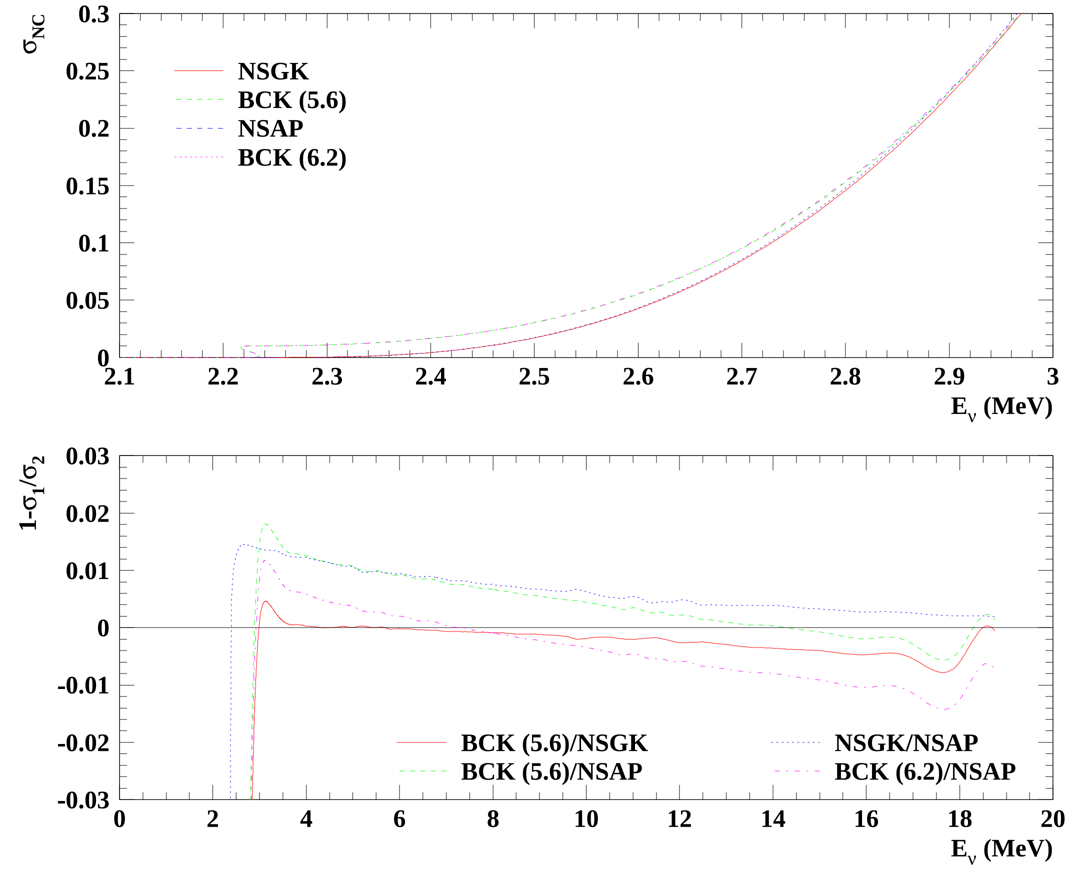

Figure 6: The top plot shows a zoom in on the neutral current total cross section at neutrino energies (E_{nu}) between 2.1 and 3.0 MeV for the five different cross section calculations. The bottom plot shows (1.0-ratios) of the BCK, NSGK and NETAL cross sections.

the updated analysis methods in the NETAL paper.

Anti-Neutral Current Cross Sections

Figure 8 shows the NC cross sections for the KN and NETAL cross sections in SNOMAN. The bottom plot shows the agreement between the two calculations, and it is observed that the two calculations agree within 1% over the entire energy range.

SNOMAN Implementation of Anti-neutrino Cross Sections

The anti-neutrino cross sections can be used within SNOMAN in a similar manner as for the regular cross sections. The main difference is with respect to the anti-CC events which contain three particles in the final state. A new option has been added into the MMSW titles file which can activate the anti-neutrino cross sections, it is

```
SET BANK MMSW 1 WORD 25 TO X
```

where X is equal to zero by default, ie. use regular cross sections. If X is set to 1, then the anti-neutrino cross sections are used. The remaining MMSW titles words remain the

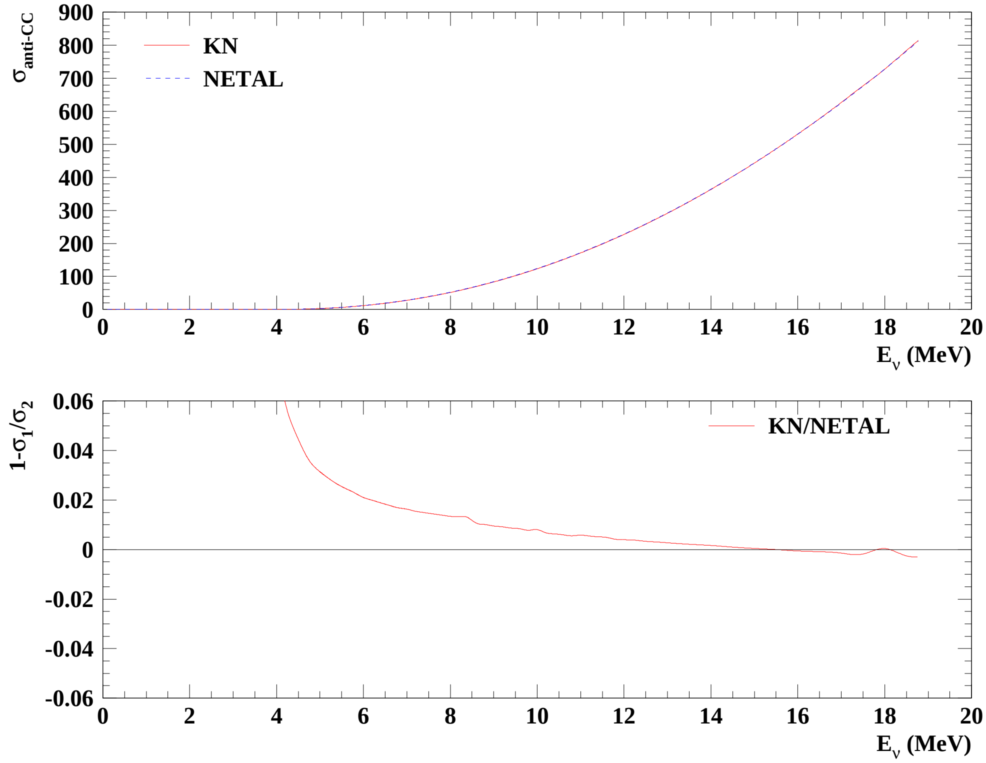

Figure 7: The anti-CC total cross section with respect to neutrino energy (E_{nu}) for the KN and NETAL cross section calculations.

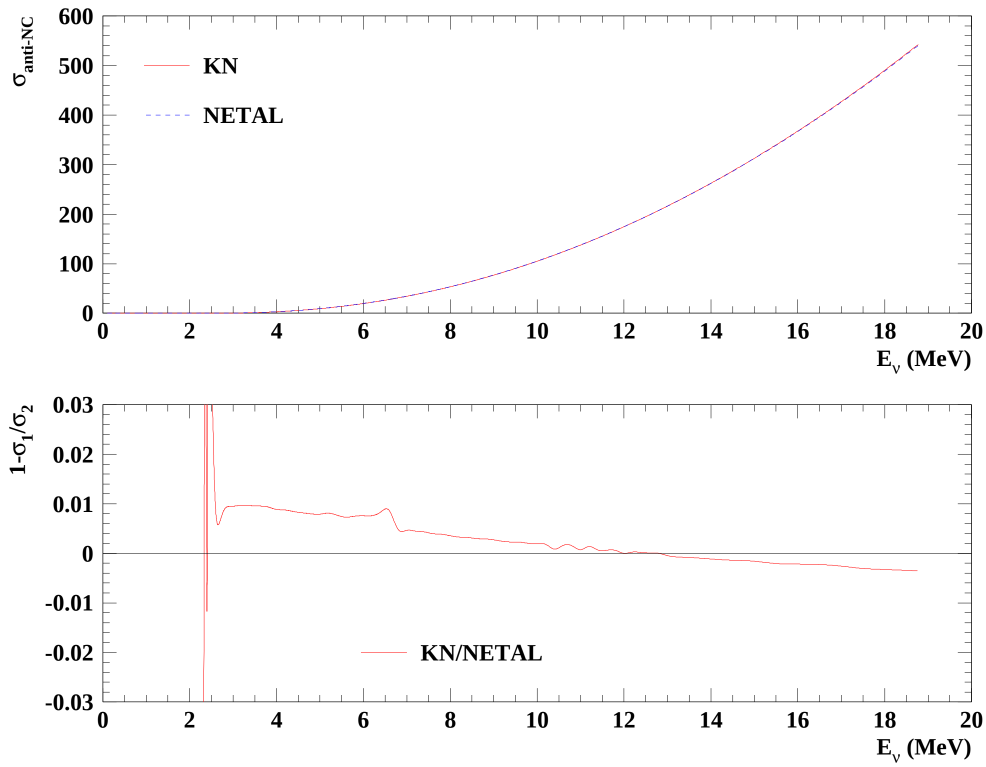

Figure 8: The anti-NC total cross sections with respect to neutrino energy (E_{nu}) for the KN and NETAL cross section calculations.

same. You must run anti-CC events independent of anti-NC or anti-ES events since the SNOMAN generation command files are different for the reactions. To generate anti-ES or NC events one can simply use a modified version of gen_msw.cmd (found in the SNOMAN prod directory), all that one needs to do is change the nu_e command into nu_e_bar, and add the above MMSW word to your run_msw.cmd file and wait for the results.

To generate anti-CC events the new generation file must contain something like:

```
$mc_num_seed_vx 3

** positron
$mc_interaction_type   $2h$$start$$e_plus$
$mc_position           $pos_region 2. $D2O$0000. 1. $ACRC_IVL$0000. 1.
$mc_direction          $dir_cc_e $time_dependent
$mc_dir_neutrino_type  $nu_e_bar$.
$mc_energy             $en_cc_e 3.511
$mc_en_neutrino_type   $nu_e_bar$.
$mc_time               $tim_solar_neutrino    0. 0. 0. 0. 0. 10. 1.
$mc_misc_find_region   $unknown

** first neutron
$mc_interaction_type2  $2h$$start$$neutron$
$mc_position2          -1
$mc_direction2         $dir_cc_e $time_dependent
$mc_dir_neutrino_type2 $nu_e_bar$.
$mc_energy2            $en_cc_e 0.0
$mc_en_neutrino_type2  $nu_e_bar$.
$mc_time2              -1
$mc_misc_find_region2  $unknown

** second neutron
$mc_interaction_type3  $2h$$start$$neutron$
$mc_position3          -1
$mc_direction3         $dir_cc_e $time_dependent
$mc_dir_neutrino_type3 $nu_e_bar$.
$mc_energy3            $en_cc_e 0.0
$mc_en_neutrino_type3  $nu_e_bar$.
$mc_time3              -1
$mc_misc_find_region3  $unknown
```

The above commands will give you the 3 particle final state that is expected. As is the case for all MSW code, an energy threshold may be indicated in the command file for illustration purposes. It is not explicity required, since the MSW code ignores this threshold. However it is left in the file so that the generation file can also be used for non-MSW event generation.

ν Energy (MeV)	NETAL/NSGK	NETAL/BCK	BCK/NSGK
3.0	0.722	0.980	0.608
3.5	0.547	0.030	0.030
4.0	0.998	0.985	0.862
5.0	0.499	0.728	0.900
6.1	0.929	0.117	0.171
8.0	0.639	0.106	0.050
9.7	0.859	0.104	0.174
10.0	0.428	0.984	0.204
10.3	0.858	0.790	0.230
12.0	0.561	0.170	0.241
12.7	0.349	0.070	0.497
14.3	0.842	0.023	0.756

Table 1: The results of the Kolmogorov Test for the different CC distributions for the studied neutrino energies.

1.1.2 Differential Cross Sections and Final State Particle Energies

CC Reaction

The CC differential cross sections calculated by integrating over $\cos\theta$ and the total energy is shown for several different input neutrino energies in Figure 9. Some of the neutrino energies are at tabulated values where one would expect very good agreement between the NSGK, BCK and NETAL differential cross sections, and indeed that is what is observed. For these plots $L_{1A} = 5.6$ fm^3. To check the interpolation between the tabulated neutrino energy points, some intermediate energies were studied. Using the Kolmogorov Test, the probabilities that the different distributions come from the same parent distribution are shown in Table 1. The final state electron kinetic energy is shown in Figure 10. Also note that all of these studies are statistically limited, agreement between the different calculations can be improved with much higher statistics. All of the plots shown are based on 1 year of Monte Carlo events.

NC Reaction

The NC differential cross sections calculated by integrating over $\cos\theta$ and the kinetic energy is shown for several different input neutrino energies in Figure 11. There is good agreement between the KN, NSGK and BCK differential cross sections for the selected energies. Using the Kolmogorov Test, the probabilities that the different distributions come from the same parent distribution are shown in Table 2. The final state neutron energies from the NC interactions is plotted in figure 12.

Anti-CC Reaction

The charged current differential cross sections calculated by integrating over $\cos\theta$ and the kinetic energy is shown for several different input neutrino energies in Figure 13. There is good agreement between the KN and NETAL differential cross sections for the selected

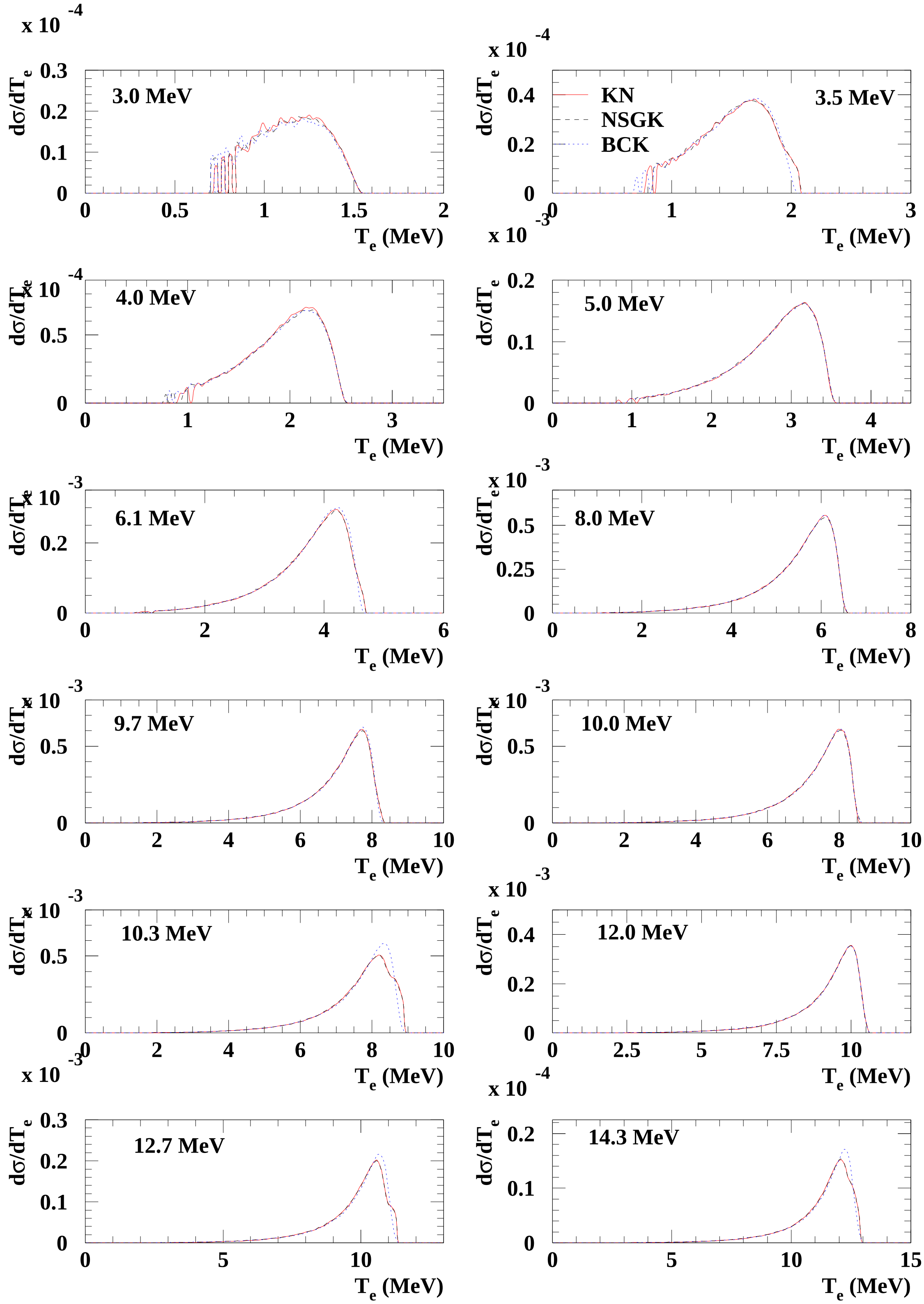

Figure 9: The NSGK, BCK and NETAL CC differential cross sections with respect to electron energy (E_e) for several different neutrino energies.

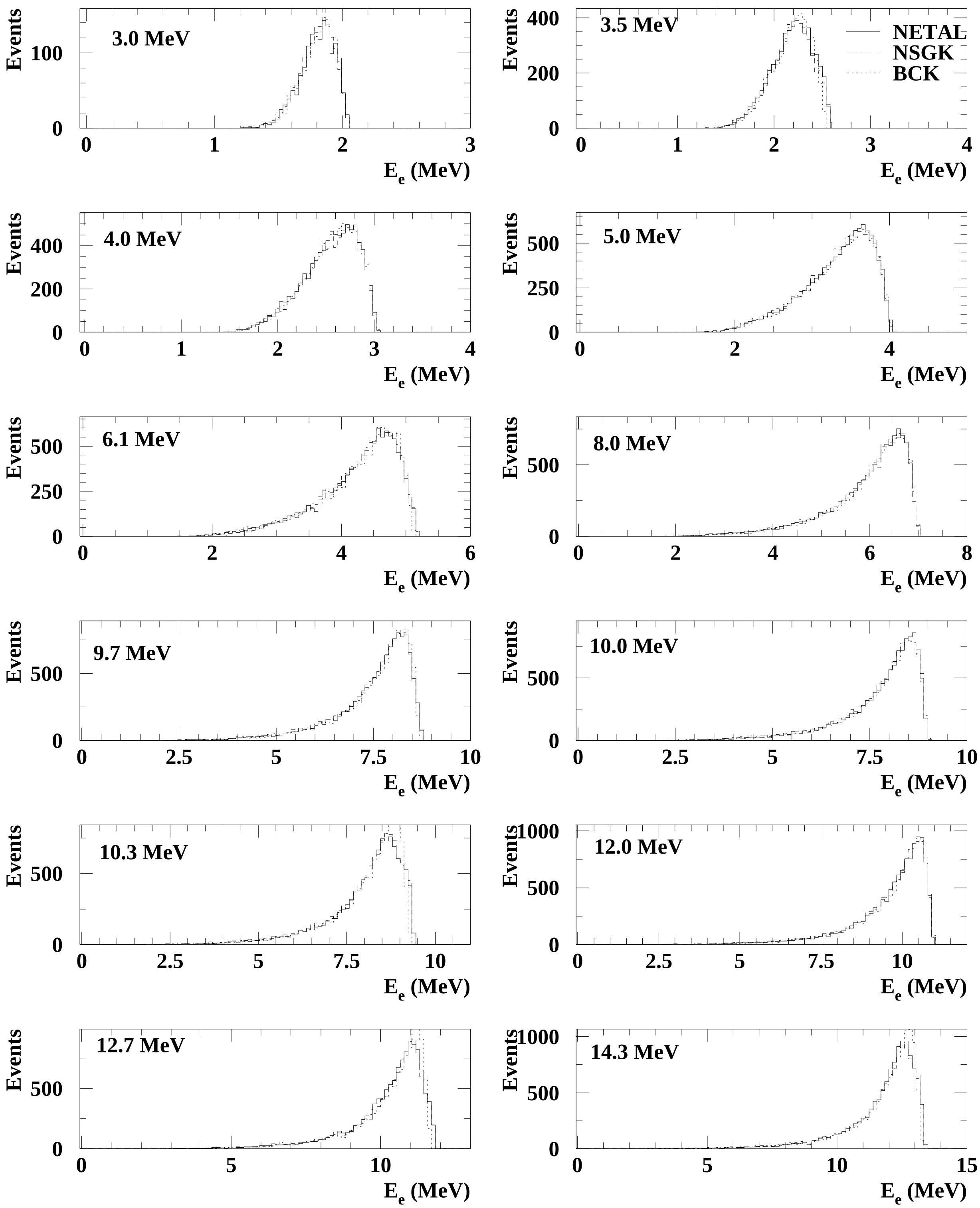

Figure 10: The NSGK, BCK and NETAL electron energy for several different neutrino energies.

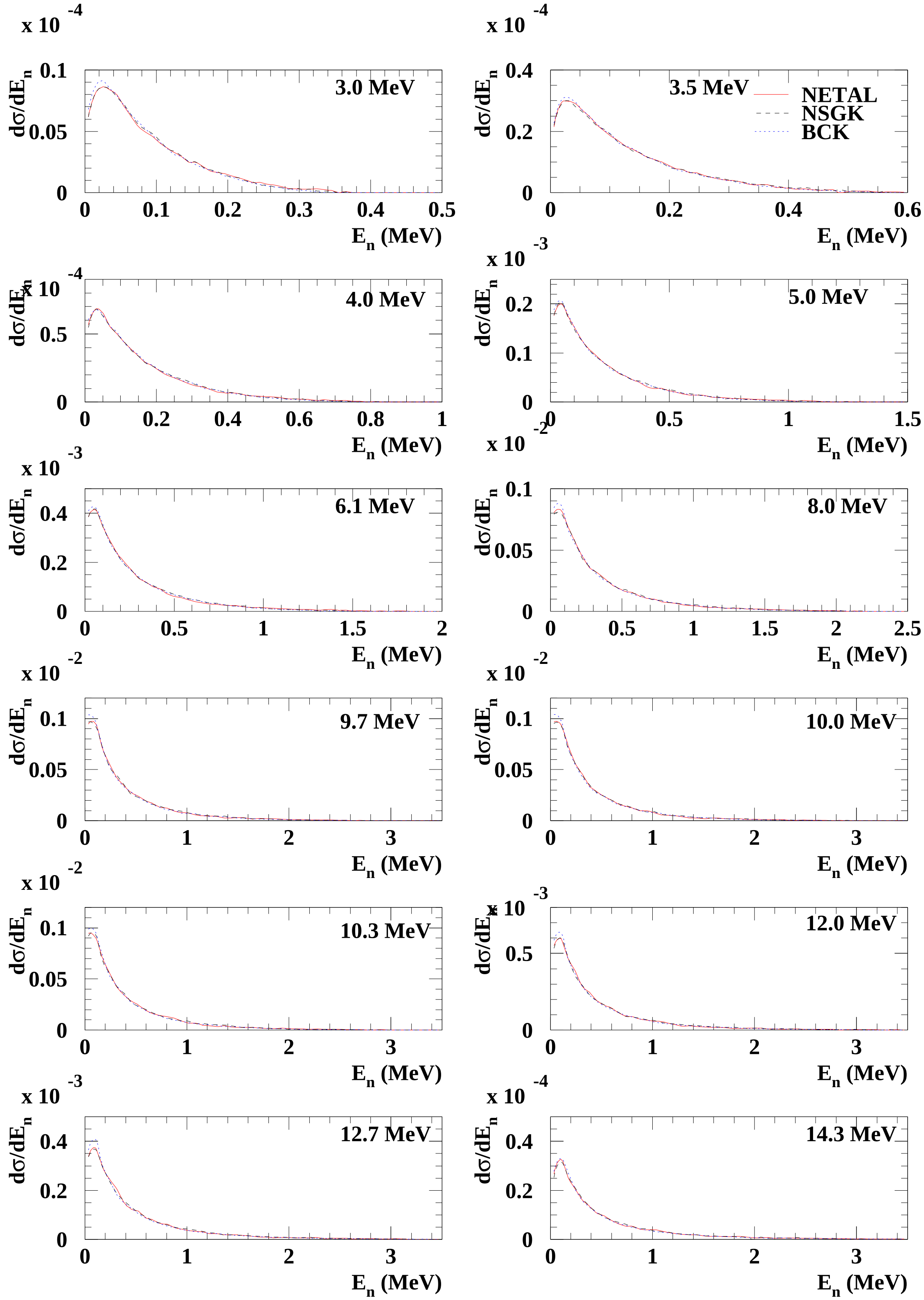

Figure 11: The KN, NSGK and BCK NC differential cross sections with respect to neutron energy (E_n) for several different neutrino energies.

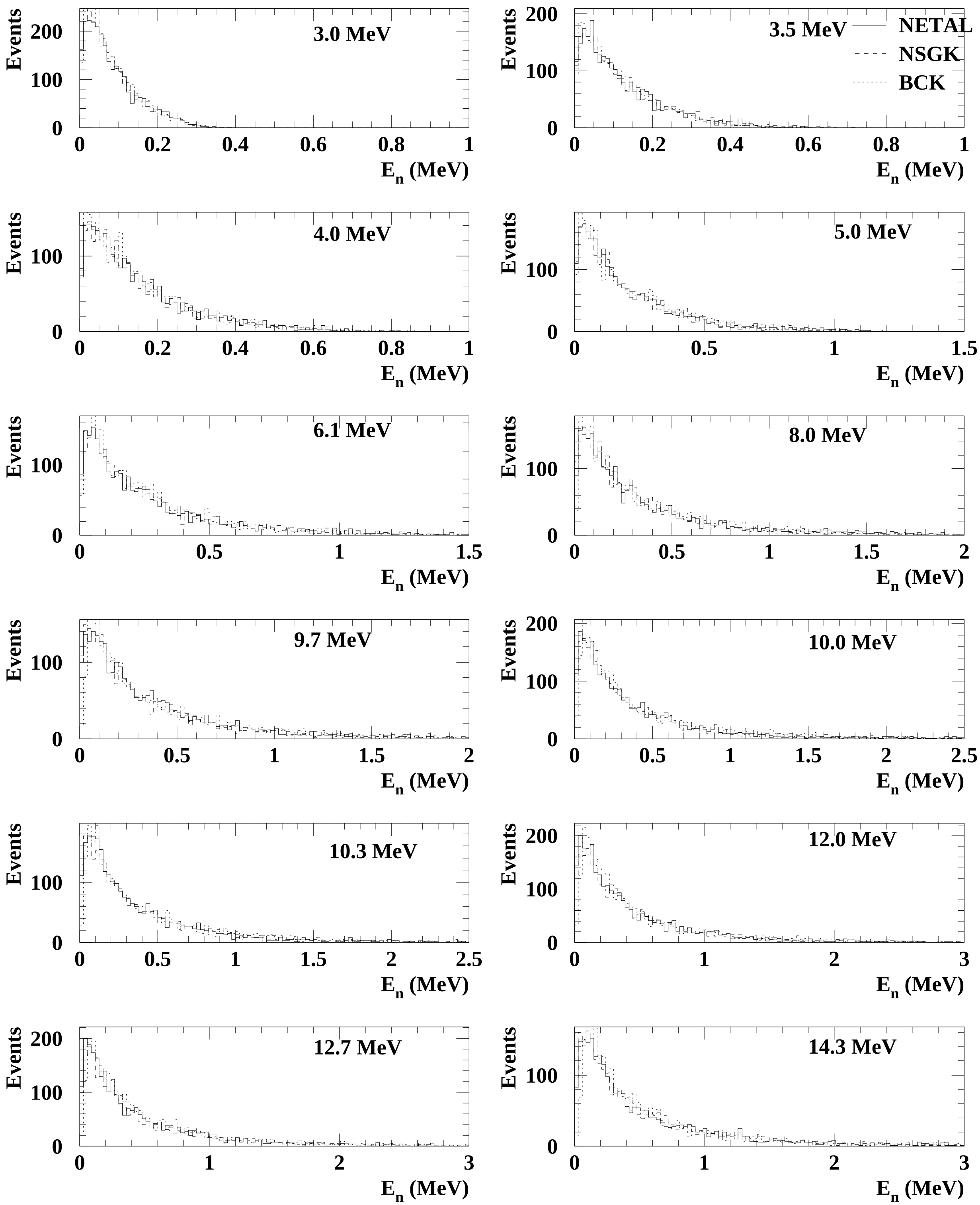

Figure 12: The NSGK, BCK and NETAL neutron energy for several different neutrino energies.

ν Energy (MeV)	NETAL/NSGK	NETAL/BCK	BCK/NSGK
3.0	0.960	0.188	0.819
3.5	0.866	0.597	0.385
4.0	0.999	0.885	0.921
5.0	0.923	0.836	0.841
6.1	0.955	0.455	0.366
8.0	0.885	0.562	0.565
9.7	0.405	0.499	0.499
10.0	0.787	0.192	0.967
10.3	0.450	0.891	0.891
12.0	0.788	0.904	0.882
12.7	0.483	0.207	0.159
14.3	0.969	0.890	0.923

Table 2: The results of the Kolmogorov Test for the different NC distributions for the studied neutrino energies.

ν Energy (MeV)	KN/NETAL
5.0	0.215
6.1	0.447
8.0	0.682
9.7	0.915
10.0	0.877
10.3	0.009
12.0	0.723
12.7	0.741
14.3	0.394

Table 3: The results of the Kolmogorov Test for the different anti-NC distributions for the studied neutrino energies.

energies. Using the Kolmogorov Test, the probabilities that the different distributions come from the same parent distribution are shown in Table 3. The final state positron kinetic energy is shown in Figure 14. In addition, the neutron energies are plotted in Figures 15 and 16. The first neutron figure shows the first neutron while the second figure shows the second generated neutron energy. Note that the second neutron is generated so that energy and momentum are conserved in the reaction.

Anti-NC Reaction

The neutral current differential cross sections calculated by integrating over $\cos\theta$ and the outgoing neutron energy is shown for several different input neutrino energies in Figure 17. There is good agreement between the KN and NETAL differential cross sections for the selected energies. The final state neutron energy is shown in Figure 18. Using the Kolmogorov Test, the probabilities that the different distributions come from the same parent

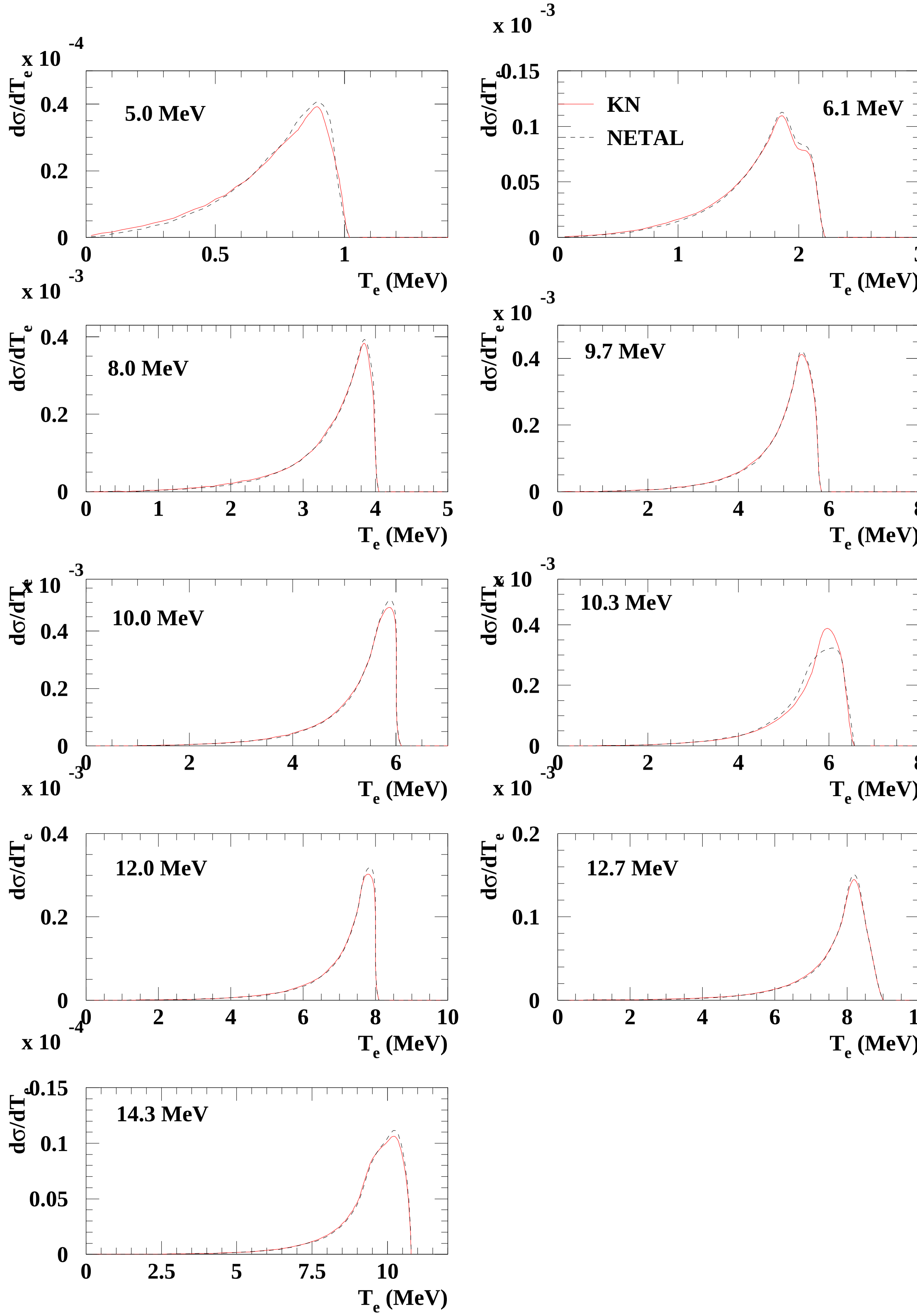

Figure 13: The KN and NETAL anti-CC differential cross sections with respect to the positron energy (E_n) for several different neutrino energies.

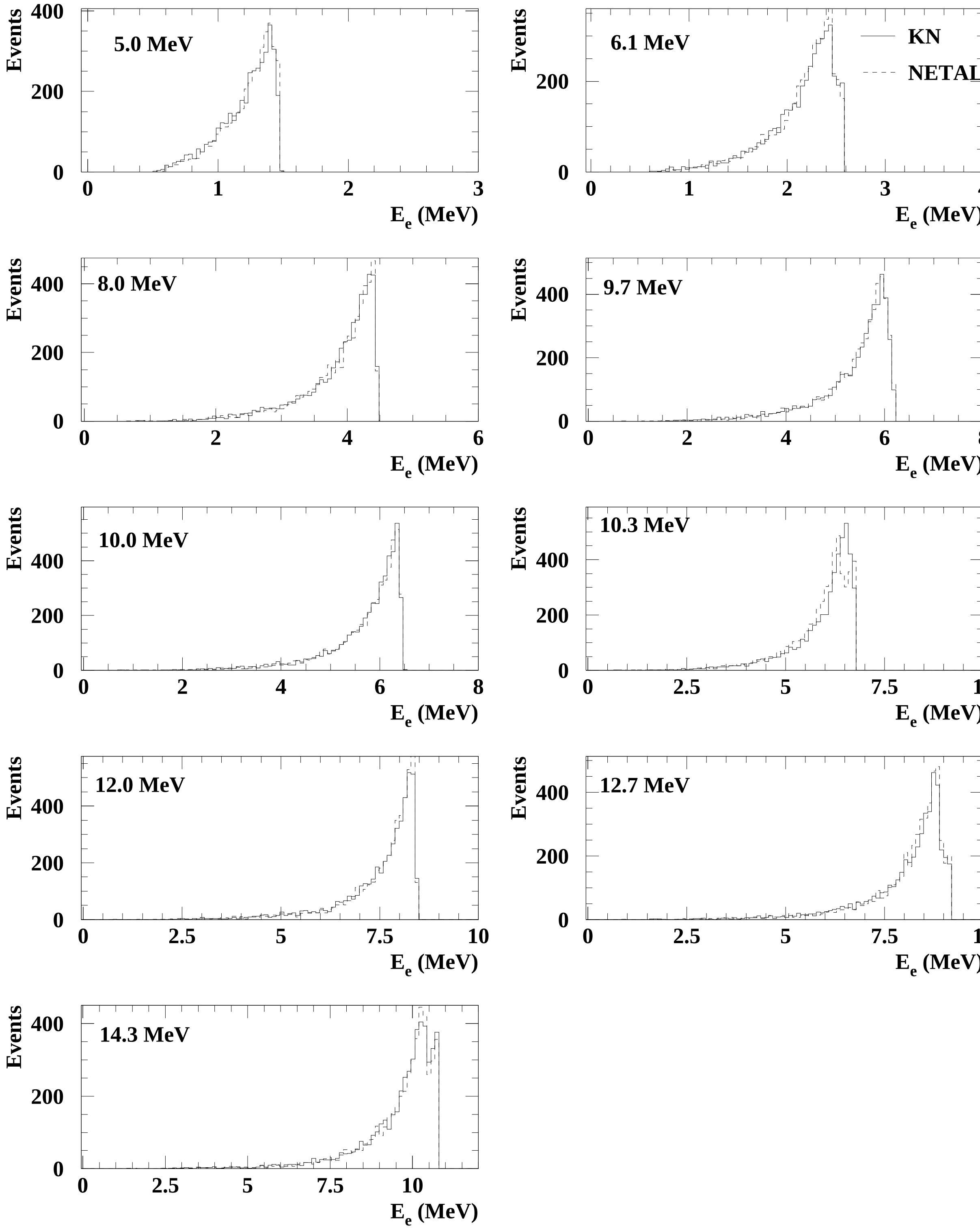

Figure 14: The KN and NETAL positron kinetic energy for several different neutrino energies.

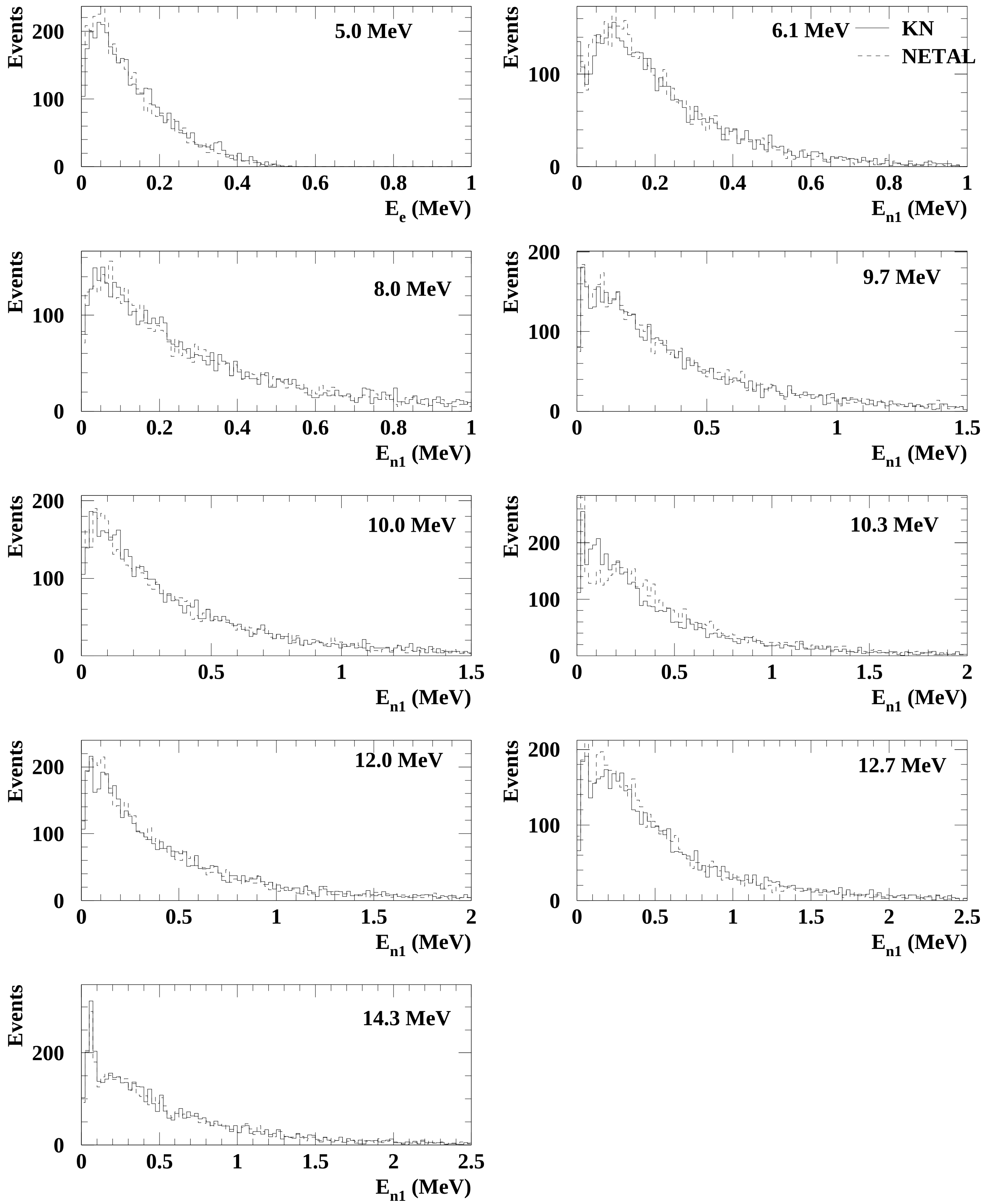

Figure 15: The KN and NETAL first neutron energy for several different neutrino energies.

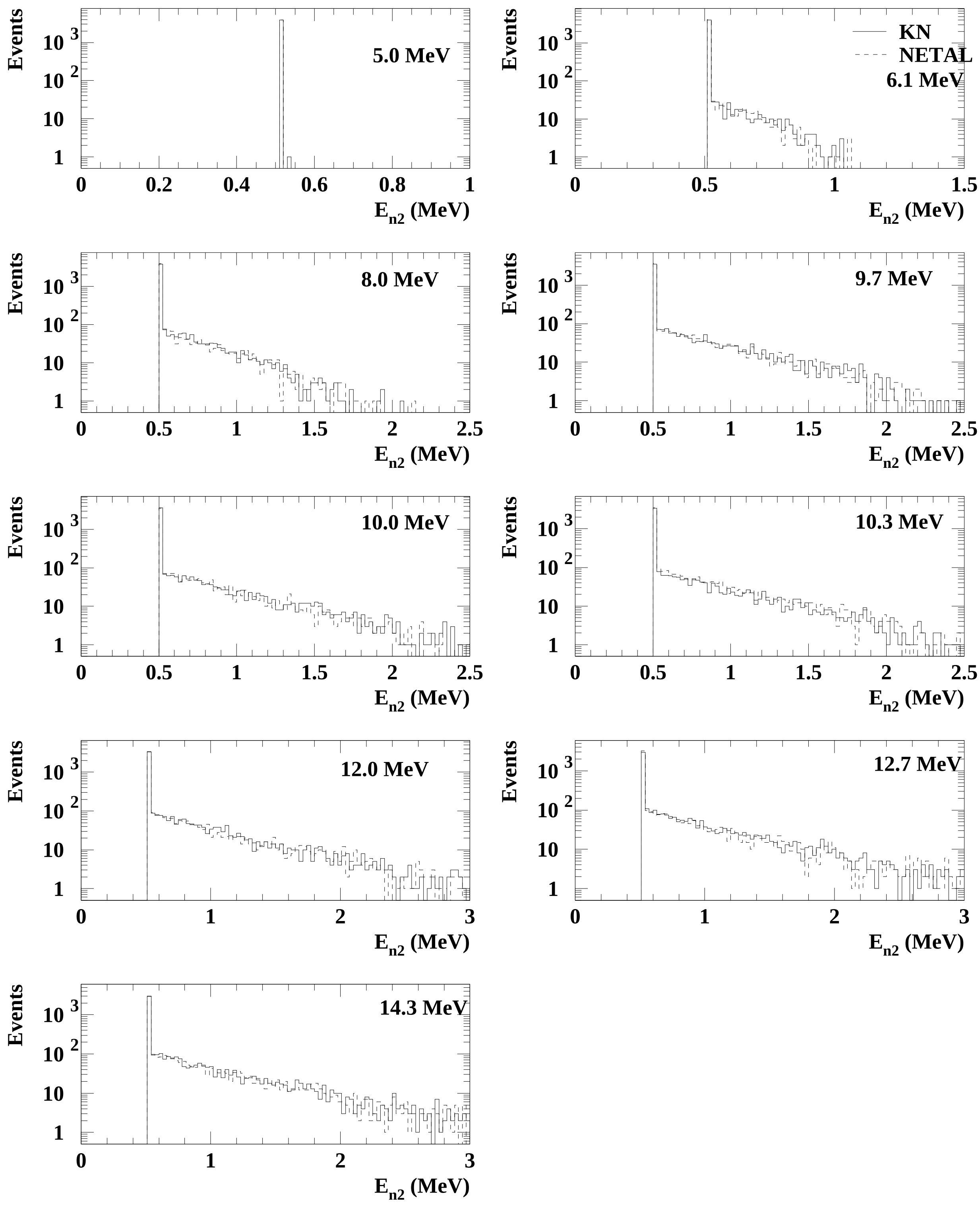

Figure 16: The KN and NETAL second neutron energy for several different neutrino energies.

ν Energy (MeV)	KN/NETAL
3.0	0.999
3.5	0.793
4.0	0.601
5.0	0.727
6.1	0.819
8.0	0.999
9.7	0.783
10.0	0.790
10.3	0.910
12.0	0.838
12.7	0.769
14.3	0.434

Table 4: The results of the Kolmogorov Test for the different anti-NC distributions for the studied neutrino energies.

distribution are shown in Table 4.

2 Summary

In summary, the newest Kubodera (NETAL) cross sections have been added into SNOMAN as titles files. The cross sections have been checked against previous calculations and they agree within 1%. This agreement includes both νd and $\overline{\nu}$d reactions. The NETAL cross sections include some of the radiative corrections, but not all of them. To include all of the corrections to the νd reactions one should continue to use the BCK or NSGK cross sections and then apply the radiative correction routines. The best L_{1A} factor was estimated to be 6.2 fm^3 for the new NETAL cross sections. Finally, there was good agreement between the differential cross sections and output particle energies for each of the reactions.

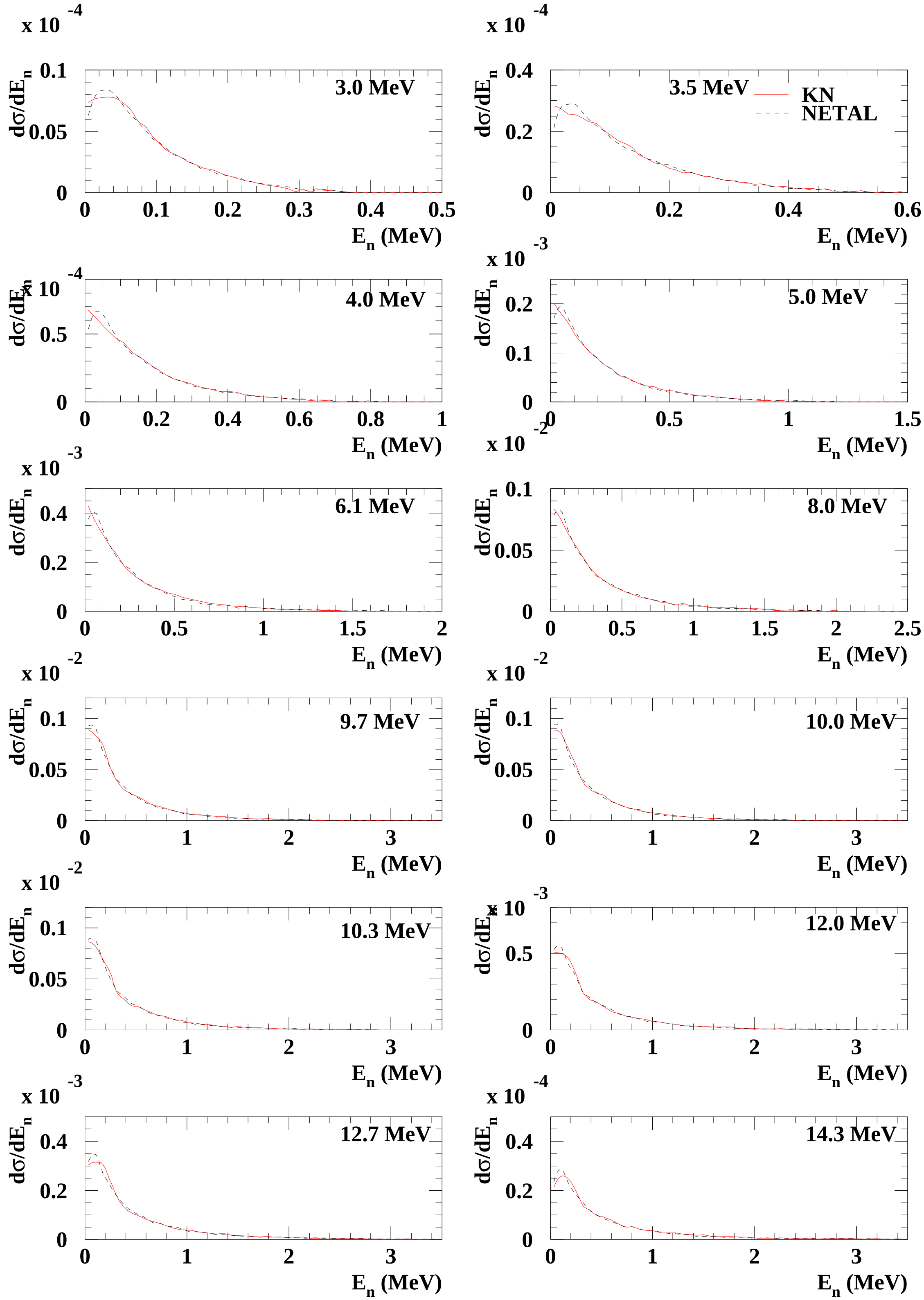

Figure 17: The KN and NETAL anti-NC differential cross sections with respect to neutron energy (E_n) for several different neutrino energies.

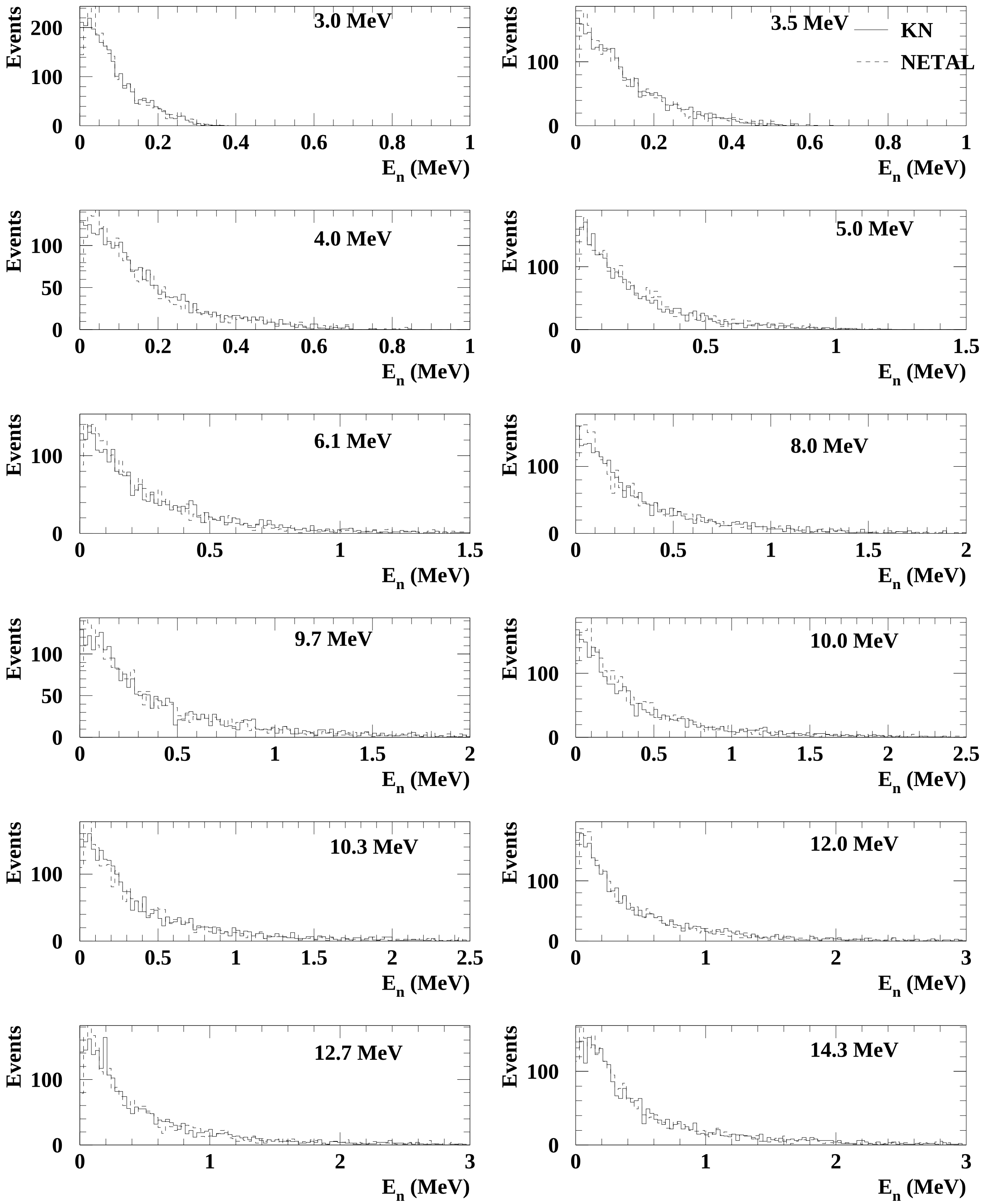

Figure 18: The KN and NETAL neutron energy for several different neutrino energies.

References

[1] J. N. Bachall *Neutrino Astrophysics* Cambridge University Press, Cambridge UK (1989) Chapter 8.

[2] K. Kubodera and S. Nozawa, Int. J. Mod. Phys. **E3** (1994) 101.

[3] S. Ying, W.C. Haxton and E.M. Henley, Phys. Rev. **C45** (1992) 1992.

[4] S. Nakamura, T. Sato, V. Gudkov and K. Kubodera, Phys. Rev. **C63** (2001) 034617.

[5] M. Butler, J-W Chen and X. Kong, Phys. Rev. **C63** (2001) 035501.

[6] J. Bahcall and E. Lisi, Phys. Rev. **D54** (1996) 5417;
J. Ellis and J. Bahcall, Nucl. Phys. **A114** (1968) 636.

[7] S. Nakamura *et al.*, nucl-th/0201062.

[8] A. Kurylov, M.J. Ramsy-Musolf and P. Vogel, nucl-th/0110051.

[9] J. Formagio, *Radiative Corrections in SNOMAN*, March 12, 2002.

[10] H. Robertson, *Radiative Corrections to the Neutrino-Deuteron Cross Sections*, April 16, 2002.